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NONLINEAR OCEAN INTERNAL WAVES[*][†]

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Abstract

In this paper, several important mathematical models of nonlinear ocean internal waves are introduced, and the related results are given.

Keywords nonlinear ocean internal waves; mathematical model
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1 Introduction

Besides the sea level fluctuation, the fluctuation phenomenon also occurs in the ocean, called ocean internal waves. It is a fluctuation of structural stability occurs in the density layer of seawater. The amplitude of ocean internal waves reaches its maximum below the sea level.

Internal wave is an important form of seawater motion the same as the surface wave in the ocean. It transmits energy from large- and meso-scale motion to small-scale motion, which leads to the internal mixing of seawater. Also it carries the deep and cold water along with its nutrients to the upper layer of the ocean, which is good for the growth of marine organisms. The wave of isodense surface caused by internal wave will affect the magnitude and direction of sound velocity in the ocean, thus affect the effectiveness of sonar, and play a beneficial or harmful role in submarine concealment and monitoring. The ups and downs of isodense surface of the ocean will make the underwater submarine navigate and stay up and down. In particular, nonlinear internal waves have great energy. They interact with other objects and have their own nonlinear effects, which can destroy ships, submarines and aircraft carriers to a great extent. Therefore, the research on it is not only of academic significance, but also of great practical and military significance.

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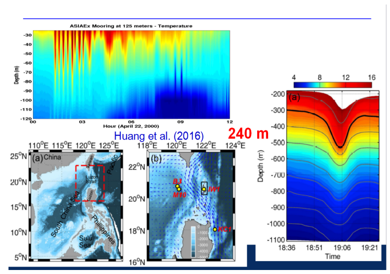

1.1 Interfacial waves

Internal waves in the ocean have many properties that surface waves don't have. Interfacial waves is one of the simplest forms of internal waves called interfacial waves, which occur at the interface of two layers of seawater with different densities. The density in the real ocean varies continuously. The waves among the strong spring layers in the ocean can be regarded as interfacial waves so that we can explain lots of internal wave phenomena

$$c = \left\{ \frac{g\lambda(\rho_2 - \rho_1)}{2\pi[\rho_2 \mathrm{cth}(\frac{2\pi h_2}{\lambda}) + \rho_1 \mathrm{cth}(\frac{2\pi h_1}{\lambda})]} \right\}^{\frac{1}{2}} \tag{1.1}$$

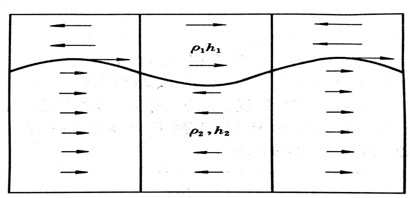

Figure 1.1: Interfacial waves

(1) The interfacial shortwaves

When the interface is in the middle of an ocean of infinite depth, we have $\lambda \ll h_1, h_2$. From (1.1), we obtain

$$c = \left[\frac{g\lambda(\rho_2 - \rho_1)}{2\pi(\rho_2 + \rho_1)} \right]^{\frac{1}{2}}. \tag{1.2}$$

(2) The interfacial longwaves

When $\lambda \gg h_1, h_2$, from (1.1), we obtain

$$c = \left[\frac{g h_1 h_2(\rho_2 - \rho_1)}{(h_1 + h_2)\overline{\rho}} \right]^{\frac{1}{2}}, \tag{1.3}$$

where $\overline{\rho} = (\rho_2 + \rho_1)/2$. Thus the formula of interfacial wave contains coefficients of $[(\rho_2 - \rho_1)/(\rho_2 + \rho_1)]^{1/2}$, which is different from that of surface wave. There is a small density contrast between the two layers in the oceans, so the coefficient is also very