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THE BOUNDS ABOUT THE WHEEL-WHEEL RAMSEY NUMBERS*

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Abstract

In this paper, we determine the bounds about Ramsey number $R(W_m, W_n)$, where W_i is a graph obtained from a cycle C_i and an additional vertex by joining it to every vertex of the cycle C_i. We prove that $3m+1 \leq R(W_m, W_n) \leq 8m-3$ for odd n, $m \geq n \geq 3$, $m \geq 5$, and $2m+1 \leq R(W_m, W_n) \leq 7m-2$ for even n and $m \geq n+502$. Especially, if m is sufficiently large and $n = 3$, we have $R(W_m, W_3) = 3m+1$.

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1 Introduction

Throughout the paper, all graphs considered are undirected, finite and contain neither loops nor multiple edges. For given graphs G, H, the Ramsey number, denoted by $R(G, H)$, is defined to be the smallest integer N such that in any edge-coloring of complete graph K_N by colors red and blue, there exists either a red G or a blue H. A wheel W_m is a graph obtained from C_m and an additional vertex by joining it to every vertex of C_m. For a graph H and a vertex $x \in H$, define $N_H(x)$ as the subgraph induced by all vertices adjacent to x in H, and $c(H), g(H)$ denote the lengths of a longest and shortest cycle of H. A graph H is called weakly pancyclic if it contains cycles of every length between $g(H)$ and $c(H)$. Let $\chi(H)$ be the chromatic number of H, that is, the smallest number needed to color the vertices of H so that no pair of adjacent vertices have the same color, and $\sigma(H)$ be the size of the smallest color class among all proper $\chi(H)$-colorings of H.

There is a famous lower bound of $R(G, H)$ observed by Burr [3] as follows

$$R(G, H) \geq (\chi(H)-1)(|G|-1)+\sigma(H).$$

If $R(G, H)$ is equal to this bound, we say that G is H-good.

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For Ramsey numbers about wheels, Surahmat et al. [10] proved that F_n is W_3-good for $n \geq 3$ where F_n consists of n triangles sharing exactly one common vertex. Hendry calculated $R(W_3, W_4) = 17$ in [7] and $R(W_4, W_4) = 15$ in [8]. Faudree and Mckay [6] proved the value of $R(W_m, W_5)$ for $m = 3, 4, 5$, and $R(W_5, W_3) = 19$. Yanbo Zhang et al. [12] obtained the exact value of $R(F_n, W_m)$ for odd $m \geq 3$, $n \geq (5m+3)/4$ and the exact value of $R(T_n, W_m)$ for every ES-tree T_n odd $m \geq 3$, $n \geq m-2$. Also [11] proved that

$$R(W_m, W_4) = \begin{cases} 2m+3 & \text{for odd } m \geq 133, \\ 2m+2 & \text{for even } m \geq 134. \end{cases}$$

Motivated by the above works, in this paper, we shall consider the upper bound of the wheel-wheel Ramsey number $R(W_m, W_n)$. The main results are as follows.

Theorem 1 (i) *If n is odd, $m \geq n \geq 3$ and $m \geq 5$, then*

$$3m+1 \leq R(W_m, W_n) \leq 8m-3.$$

(ii) *If n is even, $m \geq n+502$, then*

$$2m+1 \leq R(W_m, W_n) \leq 7m-2.$$

2 The Preliminary Lemmas

In order to establish the main results, we introduce some useful lemmas at first.

Lemma 1[4] *Every nonbipartite graph G of order n with $\delta(G) \geq (n+2)/3$ is weakly pancyclic with $g(G) = 3$ or 4.*

Lemma 2[1] *Let G be a graph with $\delta(G) \geq 2$. Then $c(G) \geq \delta(G)+1$. Moreover, if $\delta(G) \geq |G|/2$, then G has a Hamilton cycle.*

Lemma 3[2] *$R(W_m, C_3) = 2m+1$ for $m \geq 5$.*

Lemma 4[5] *$R(C_m, W_n) = 3m-2$ for odd n, $m \geq n \geq 3$ and $(m,n) \neq (3,3)$.*

Lemma 5[13] *$R(C_m, W_n) = 2m-1$ for even n and $m \geq n+502$.*

Lemma 6[9] *For all $p \geq 3$, $q \geq 1$, $0 < \gamma < 1$, there exist $c > 0$, $\eta > 0$ such that if n is large and $E(K_{p(n-1)+1}) = E(R) \cup E(B)$ is a 2-coloring, then one of the following statements holds:*

(i) *R contains $K_{p+1}(1, 1, t, \cdots, t)$ for $t = \lceil clogn \rceil$;*

(ii) *B contains every q-degenerate, (γ, η)-splittable graph G of order n.*

We recall that a graph G is called q-degenerate if each of its subgraphs contains a vertex of degree at most q, that is, $q = \max\{\min\{d(u), u \in V(G')\}, G' \in \mathscr{G}\}$ where $\mathscr{G}$ is the set of all subgraphs of G. For given real numbers $\gamma, \eta > 0$, we say that the graph G of order n is (γ, η)-splittable if there exists a set $S \subseteq V(G)$ with $|S| < n^{1-\gamma}$ such that the order of any component of $G - S$ is at most ηn.