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ASYMPTOTIC BEHAVIOR OF WAVE EQUATION OF KIRCHHOFF TYPE WITH STRONG DAMPING*†

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Abstract

The paper deals with the strongly damped nonlinear wave equation of Kirchhoff type. The existence of a global attractor is proven by using the decomposition, and moreover, the structure of the global attractor is established. Our results improve the previous results.

Keywords wave equations; global attractors; critical nonlinearity
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1 Introduction

The nonlinear evolution equations have been investigated by many authors. We consider the following problem

$$\begin{aligned} &u_{tt} - M(\|\nabla u\|^2)\triangle u - \triangle u_t + f(u_t) + g(u) = h(x), \quad x \in \Omega,\ t > 0, \\ &u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), \quad x \in \Omega,\ u|_{\partial\Omega} = 0, \end{aligned} \tag{1}$$

where $M(s) = 1 + s^{\frac{m}{2}},\ m \geq 2$, $\Omega \subset \mathbf{R^3}$ is a bounded domain with smooth boundary $\partial\Omega$. The assumptions on $f(u_t)$, $g(u)$ and $h(x)$ will be specified below.

When $N = 1$, such an equation without the dissipative term $\triangle u_t$ is introduced to describe the vibration of an elastic string. The original equation is

$$\rho h u_{tt} + \delta u_t = \Big\{p_0 + \frac{Eh}{2L}\int_0^L \Big(\frac{\partial u}{\partial x}\Big)^2 \mathrm{d}x\Big\}\frac{\partial^2 u}{\partial x^2} + f,$$

for $0 < x < L,\ t \geq 0$, where $u = u(x,t)$ is the lateral displacement at the space coor-

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dinate x and the time t, E is the Young modulus, ρ is the mass density, h is the cross-section area, L is the length, p_0 is the initial axial tension, δ is the resistance modulus, and f is the external force. When $\delta = f = 0$, the equation is firstly introduced by Kirchhoff [6].

Equation (1) is also mathematically interesting and has been extensively investigated by many authors. By using asymptotic compactness the authors dealt with some absorbing properties of global attractor of the Kirchhoff type equation

$$u_{tt} - M(\|\nabla u\|^2)\triangle u - \triangle u_t + g(x,u) = h(x),$$

where g does not exhibit a critical growth [8].

The paper [13] studied the longtime behavior of the Kirchhoff type equation

$$u_{tt} - M(\|\nabla u\|^2)\triangle u - \triangle u_t + u + u_t + g(x,u) = h(x)$$

on R^n. It showed that the related continuous semigroup possesses a global attractor which is connected and has finite fractal and Hausdorff dimensions.

In [9] by using two half invariant sets, the author proved the existence and some absorbing properties of an attractor in a local sense for the initial boundary value problem of a quasilinear wave equation of Kirchhoff type

$$u_{tt} - (1 + \|\nabla u\|_2^2)\triangle u + u_t + g(x,u) = h(x).$$

Nonlinear evolution equations have been investigated by many authors, see [1-5,8-14], but, there are relatively few results on the global attractor for problem (1), where the functions f and g exhibit a critical growth. The problem considered in this manuscript is more difficult to be dealt with than those considered in [8,9] because the difficulty is caused not only by the critical growth of f and g but also by the nonlinearity of M. The aim of this paper is to improve the main results of [8,9], that is, by utilizing the decomposition idea [10] we prove the existence of a global attractor of (1). Still, the structure of the global attractor is established.

2 Preliminary

We first introduce the following notations:

$$L^p = L^p(\Omega), \quad H^k = H^k(\Omega), \quad H_0^1 = H_0^1(\Omega), \quad \|\cdot\|_p = \|\cdot\|_{L^p}, \quad \|\cdot\| = \|\cdot\|_{L^2},$$

with $p \geq 1$. The notations $(\cdot,\cdot)$ and $[\cdot,\cdot]$ will be used as the L^2-inner product and the duality pairing between dual spaces respectively. For brevity, we use the same letter C to denote different positive constants, and $C(\cdots)$ to denote positive constants depending on the quantities appearing in the parenthesis. In L^2 we introduce the operator $-\triangle$ with the domain $D(-\triangle) = H^2 \cap H_0^1$, where $-\triangle$ is the Laplace operator in Ω with the Dirichlet boundary condition. Below we denote by e_k the orthonormal