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RE-WEIGHTED NADARAYA-WATSON ESTIMATION OF CONDITIONAL DENSITY*†

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Abstract

In order to avoid the discussion of equation (1.1), this paper employs the proof method of Liang (2012) to consider the re-weighted Nadaraya-Watson estimation of conditional density. The established results generalize those of De Gooijer and Zerom (2003). In addition, this paper improves the bandwidth condition of Liang (2012).

Keywords re-weighted Nadaraya-Watson estimation; conditional density; bandwidth condition

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1 Introduction

Re-weighted Nadaraya-Watson (RNW) method is a weighted version of Nadaraya-Watson (NW). The RNW estimator of the conditional distribution function was proposed by Hall, Wolff and Yao (1999). Later, Cai (2001) applied the RNW method to the estimation of the conditional mean function including the conditional distribution function. The results in Cai (2001) show that the RNW estimation not only possesses the bias of local linear (LL) estimator, but also preserves the property of NW estimator: the estimated values of the conditional mean function are always within the range of the response variable. In the case of estimating a positive quantity such as conditional distribution, the LL method may assign negative weights to certain sample points, and the corresponding LL estimator may produce a negative result in finite samples, thus lead to unreasonable inference. In this case, the RNW estimator works better because it is guaranteed to be nonnegative in finite samples and also has the good bias of the LL estimator. The RNW method has been applied to estimate conditional density (see De Gooijer and Zerom (2003)), to estimate the

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volatility function of some diffusion models (see Xu (2010), Hanif, Wang and Lin (2012), Wang, Zhang and Tang (2012), Song, Lin and Wang (2013)), to estimate the conditional variance function (see Xu and Phillips (2011)).

However, in the beginning of proofs of their theorems in Cai (2001) and De Gooijer and Zerom (2003), they used the following equation

$$\sum_{i=1}^{n}\{1+X_{ni}\}Y_{ni}=\{1+o_p(1)\}\sum_{i=1}^{n}Y_{ni}, \quad \text{that is,} \quad \sum_{i=1}^{n}X_{ni}Y_{ni}=o_p(1)\sum_{i=1}^{n}Y_{ni}, \tag{1.1}$$

where $\{(X_{ni},Y_{ni})|\,1\le i\le n,\, n\ge 1\}$ is an $\mathbb{R}\times\mathbb{R}$ valued stationary process and $\max\limits_{1\le i\le n}|X_{ni}|=o_p(1)$, (1.1) holds in the case that $\{Y_{ni}\ge 0|\,1\le i\le n\}$ or $\{Y_{ni}\le 0|\,1\le i\le n\}$, while (1.1) may be discussed in other cases including $\sum\limits_{i=1}^{n}Y_{ni}=o_p(1)$. Recently, Liang (2012) extended the RNW method of Cai (2001) to the conditional mean function with left-truncated and dependent data, where the author employed another method which can avoid the discussion of equation (1.1) to prove the main result. And in the case of no left truncation, the results are the same to those of Cai (2001). Can the proof method of Liang (2012) be used to consider the RNW estimator of conditional density but avoid the discussion of equation (1.1)?

In this paper, we apply the analysis approach of Liang (2012) to consider the RNW estimator of conditional density. The RNW estimator here is the generalization of that of De Gooijer and Zerom (2003) since it uses different kernel functions and bandwidths in both directions. The established results generalize those of De Gooijer and Zerom (2003). The contributions of this paper are two fold. First, this paper can avoid the discussion of (1.1) in the proof of the results. Second, this paper improves the bandwidth condition in Liang (2012). The rest of the paper is organized as follows. Section 2 introduces the RNW estimator. Assumptions and the main results are stated in Section 3. Section 4 is devoted to proving the main results.

2 Model and RNW Estimator

2.1 Model

Let $\{(X_i,Y_i),\, i\ge 1\}$ be an $\mathbb{R}\times\mathbb{R}$ valued, strictly stationary and α mixing process with a common probability density function $f(\cdot,\cdot)$ as (X,Y). Assume that X admits a marginal density $g(\cdot)$. Of interest is estimating of the conditional density of Y given $X=x$, that is,

$$f(y|x)=\frac{f(x,y)}{g(x)}, \quad y\in\mathbb{R}, \tag{2.1}$$

where $g(x)>0$.