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STABILITY PROPERTY OF A PREDATOR-PREY SYSTEM WITH A CONSTANT PROPORTION OF PREY REFUGE AND STAGE-STRUCTURE FOR PREY SPECIES*†

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Abstract

A predator-prey system with a constant proportion of prey refuge and stage-structure for prey species is proposed and studied in this paper. A set of conditions for the permanence of the system is obtained. The local stability of the system is discussed by the sign of eigenvalues. Furthermore, by using the iterative method, some suitable sufficient conditions for the global attractivity of the interior equilibrium is obtained. Our study shows that the constant proportion of prey refuge could lead to more complicate dynamic behaviors. Numerical simulations are also presented to illustrate the feasibility of the main results.

Keywords predator-prey; stage-structure; refuge; global stability
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1 Introduction

Since the pioneer work of Aiello and Freedman [1] on the single species stage-structured models, many scholars had done excellent works on the stage-structure population dynamics. For example, Lin, Xie and Chen [10] studied the convergences of a stage-structured predator-prey model with modified Leslie-Gower and Holling-type II schemes. Their study indicated that both the stage-structure and the death rate of the mature prey play important roles on the permanence or extinction of the system.

The existence of refuges sometimes plays an important role in the co-existence of predator and prey species. A prey refuge can be broadly defined as including any

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strategies to reduce the risk of predation, such as spatial or temporal refuges, prey aggregations, or reducing prey activity. As was pointed out by Devi [11], there are t-wo types of refuges: refuges protecting a fixed number of prey and refuges protecting a constant proportion of prey. Chen et al. [12] proposed a Leslie-Gower predator-prey model incorporating a constant proportion of prey refuge. Their results showed that refuge leads to more complicate dynamic behaviors.

There are many works on stage-structure population dynamics [1-20], and many works on the predator prey system incorporate the prey refuge [11-14]. However, only recently, Devi [11] proposed a stage-structured predator-prey model with prey refuge. They considered the refuges protecting a fixed number of prey. Devi showed that the equilibrium value of mature prey population increases with the prey refuges, whereas the equilibrium value of predator population decreases with the prey refuges. The success of Devi [11] motivates us to consider the influence of a constant proportion of prey refuge and to propose the following model:

$$\begin{aligned}
&\dot{x}_i(t) = \alpha x_m(t) - \gamma x_i(t) - \alpha \mathrm{e}^{-\gamma\tau} x_m(t-\tau),\\
&\dot{x}_m(t) = \alpha \mathrm{e}^{-\gamma\tau} x_m(t-\tau) - \beta x_m^2(t) - \frac{(1-p)x_m(t)y(t)}{(1-p)x_m(t)+y(t)},\\
&\dot{y}(t) = k\frac{(1-p)x_m(t)y(t)}{(1-p)x_m(t)+y(t)} - dy(t), \qquad (1.1)\\
&x_m(\theta) = \phi_m(\theta) \geq 0, \quad -\tau \leq \theta < 0, \quad x_i(0) > 0, \quad x_m(0) > 0, \quad y(0) > 0,
\end{aligned}$$

where $x_i(t)$ and $x_m(t)$ represent the densities of the immature and mature prey populations, respectively. $y(t)$ is described as the density of predator population at time t.

System (1.1) satisfies the following assumptions:

(1) The per capita birth rate of the immature popular is $\alpha > 0$. The per capita death rate of the immature popular is $\gamma > 0$. The per capita death rate of the mature prey is proportional to the current mature prey population with a proportionality constant $\beta > 0$. $\tau > 0$ is the length of time from birth to maturity. $\mathrm{e}^{-\gamma\tau}$ denotes the surviving rate of immaturity to reach maturity. The term $\alpha\mathrm{e}^{-\gamma\tau}x_m(t-\tau)$ models the immature individuals who are born at time $t-\tau$ and survive and mature at time t;

(2) It is assumed that predators only feed on the mature prey. $k > 0$ is the efficiency with which predators convert consumed prey into new predators. $d > 0$ is the death rate of predators species. The mature prey using refuges are proportional to the existing population with a proportionality constant $0 < p < 1$.

For the continuity of the solutions to system (1.1), in this paper, we require

$$x_i(0) = \int_{-\tau}^{0} \alpha \mathrm{e}^{\gamma s}\phi_m(s)\mathrm{d}s.$$