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A STUDY ON PHASE-FIELD MODELS FOR BRITTLE FRACTURE

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Abstract. In the phase-field modeling of brittle fracture, anisotropic constitutive assumptions for the degradation of stored elastic energy due to fracture are crucial to preventing cracking in compression and obtaining physically sound numerical solutions. Three energy decomposition models, the spectral decomposition, the volumetric-deviatoric split, and a modified volumetric-deviatoric split, and their effects on the performance of the phase-field modeling are studied. Meanwhile, anisotropic degradation of stiffness may lead to a small amount of energy remaining on crack surfaces, which violates crack boundary conditions and can cause unphysical crack openings and propagation. A simple yet effective treatment for this is proposed: define a critically damaged zone with a threshold parameter and then degrade both the active and passive energies in the zone. A dynamic mesh adaptation finite element method is employed for the numerical solution of the corresponding elasticity system. Four examples, including two benchmark ones, one with complex crack systems, and one based on an experimental setting, are considered. Numerical results show that the spectral decomposition and modified volumetric-deviatoric split models, together with the improvement treatment of crack boundary conditions, can lead to crack propagation results that are comparable with the existing computational and experimental results. It is also shown that the numerical results are not sensitive to the parameter defining the critically damaged zone.

Key words. Brittle fracture, phase-field modeling, constitutive assumption, critically damaged zone, moving mesh, finite element method.

1. Introduction

In recent years, the phase-field model for brittle fracture based on the variational approach of Francfort and Marigo [7] has become a commonly used numerical simulation technique for engineering designs because it can handle complex cracks and crack initiation and propagation more easily than other methods. The basic idea of the phase-field modeling is to describe cracks by a continuous scalar field variable d, which is used to indicate whether the material is damaged or not. This variable d depends on a parameter l describing the actual width of the smeared cracks and has a value of zero or close to zero near the cracks and one away from the cracks. There are three major advantages of the phase-field modeling for brittle fracture over other methods. Firstly, the behavior of the crack is completely determined by a coupled system of partial differential equations (PDEs) based on the energy functional. Therefore, additional calculations such as stress-intensity factors are not required to determine the crack initiation and propagation. Secondly, complex fracture networks can be easily handled since crack merging and branching do not require explicitly keeping track of fracture interfaces. Thirdly, smooth interfaces with sharp gradient can be introduced into the displacement field to avoid discontinuities.

Since it was first proposed by Bourdin et al. [5, 7], the phase-field modeling for brittle fracture has attracted considerable attention and significant progress has been made; e.g., see [1, 3, 4, 15, 18, 20, 25]. However challenges still exist. In

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phase-field modeling, constitutive assumptions for the degradation of energy due to fracture can be categorized into two groups, isotropic models and anisotropic models. In the former group, the degradation function acts on the whole stored bulk energy, which means that energy is released due to fracture in both tension and compression. Thus, crack propagation may also arise under compressive load state, which is physically unrealistic. On the other hand, in order to overcome this unphysical feature, the elastically stored energy is decomposed into active and passive parts and only the former is degraded. Two commonly used energy decomposition models have been proposed in the past. Miehe et al. [20] introduced a fully anisotropic constitutive model for the degradation of energy based on the spectral decomposition of strains with the assumption that crack evolution is induced by the positive principal strains. The other model is the unilateral contact model proposed by Amor et al. [1] that splits the strain into volumetric and deviatoric parts, with the expansive volumetric part and the total deviatoric part being degraded. Since the choice of the energy splitting controls the energy contribution in the damage evolution, different splitting models can significantly affect numerical approximations in the phase-field modeling of cracks. In this work we shall study these models plus an modified version of the unilateral contact model.

In phase-field modeling, a pre-existing crack is often modeled as a discrete discontinuity in the geometry or an induced discontinuity in the phase-field. The former has been successfully applied in phase-field models for a single initial crack. However, it is difficult to handle complex crack boundary conditions since the location of the initial crack is mesh-dependent. For the latter, an initial strain-history field is introduced to define the location of the induced crack. One of the major advantages of this treatment is that initial cracks can be placed anywhere in the domain without referring to the mesh, which makes it possible to deal with complex initial cracks. The induced crack model was first proposed by Borden et al. [3] and significant improvements have been made [3, 18, 22]. However, a small amount of energy remains in the totally damaged zone due to the anisotropic degradation of stiffness. For the induced crack model, stress remains in the interior of the initial crack and increases with external loads before the crack begins to propagate. This violates the vanishing stress condition on the crack surface and often results in unphysical crack propagation. May et al. [18] have observed that with the induced crack setting for a single notched shear test, numerical results are very different from those with discrete crack boundary conditions. To overcome this problem, Strol and Seelig [23, 24] proposed a novel treatment of crack boundary conditions in which crack orientation is taken into account so that both the positive normal stress on the crack surfaces and the shear stress along the frictionless crack surface vanish. However, the establishment of this constitutive assumption is not based on the variational approach, which makes the phase-field model more complicated to implement.

Another important issue is that the phase-field modeling approximates the original discrete problem as $l \to 0$ under the condition that $h \ll l$ or at least $h < l$, where h denotes the mesh spacing. A very fine mesh is needed to fulfill the condition when a uniform mesh is used in the computation, which increases the computational cost significantly. Moreover, cracks can propagate under continuous load. Thus, it is natural to use a dynamic mesh adaptation strategy to improve the efficiency of the simulation. In this work we use the moving mesh PDE (MMPDE) method [6, 12, 13, 14] to concentrate mesh elements around evolving cracks. The reader is