Commun. Math. Res.
doi: 10.4208/cmr.2021-0039

Vol. **39**, No. 1, pp. 136-172
February 2022

Global Weak Solutions for Compressible Navier-Stokes-Vlasov-Fokker-Planck System

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Received 30 March 2021; Accepted 2 June 2021

Dedicated to Professor C.-J. Xu on the occasion of his 65th birthday

Abstract. The one-dimensional compressible Navier-Stokes-Vlasov-Fokker-Planck system with density-dependent viscosity and drag force coefficients is investigated in the present paper. The existence, uniqueness, and regularity of global weak solution to the initial value problem for general initial data are established in spatial periodic domain. Moreover, the long time behavior of the weak solution is analyzed. It is shown that as the time grows, the distribution function of the particles converges to the global Maxwellian, and both the fluid velocity and the macroscopic velocity of the particles converge to the same speed.

AMS subject classifications: 35Q30, 35Q84, 82C40

Key words: Fluid-particle model, compressible Navier-Stokes-Vlasov-Fokker-Planck, hypoellipticity, global existence, large time behavior.

1 Introduction

Fluid-particle models have a wide range applications such as biosprays in medicine, chemical engineering, compressibility of droplets, fuel-droplets in combustion theory, pollution settling processes, and polymers to simulate the motion of particles dispersed in dense fluids [1,3,8,24,32,37,39].

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In this paper, we consider the initial value problem (IVP) for the one-dimensional compressible Navier-Stokes-Vlasov-Fokker-Planck (NS-VFP) system

$$\begin{cases} \rho_t+(\rho u)_x=0, & (1.1a)\\ (\rho u)_t+(\rho u^2)_x+\big(P(\rho)\big)_x=\big(\mu(\rho)u_x\big)_x-\int_{\mathbb{R}}\kappa(\rho)(u-v)f\,dv, & (1.1b)\\ f_t+vf_x+\big(\kappa(\rho)(u-v)f-\kappa(\rho)f_v\big)_v=0,\ \ (x,v)\in\mathbb{T}\times\mathbb{R},\ \ t>0,\ \ \mathbb{T}:=\mathbb{R}/\mathbb{Z} & (1.1c) \end{cases}$$

with the initial data

$$\big(\rho(x,0),\rho u(x,0),f(x,v,0)\big)=\big(\rho_0(x),m_0(x),f_0(x,v)\big),\quad (x,v)\in\mathbb{T}\times\mathbb{R}, \tag{1.2}$$

where $\rho=\rho(x,t)$ and $u=u(x,t)$ are the fluid density and velocity associated with the dense phase (fluid) respectively, and $f=f(x,v,t)$ denotes the distribution function associated with the dispersed phase (particles). The system (1.1) can be viewed as the compressible Navier-Stokes equations (1.1a)-(1.1b) for the fluid and the Vlasov-Fokker-Planck equation (1.1c) for the particles coupled each other through the drag force term $\kappa(\rho)(u-v)$. The pressure $P(\rho)$ and the viscosity coefficient $\mu(\rho)$ are given by

$$P(\rho)=A\rho^{\gamma},\quad \mu(\rho)=\mu_0+\mu_1\rho^{\beta}, \tag{1.3}$$

and the drag force coefficient $\kappa(\rho)$ is chosen to be

$$\kappa(\rho)=\kappa_0\rho, \tag{1.4}$$

where the constants $A,\mu_0,\mu_1,\kappa_0,\gamma$, and β satisfy

$$A>0,\quad \mu_0>0,\quad \mu_1>0,\quad \kappa_0>0,\quad \gamma>1,\quad \beta\geq 0.$$

Without loss of generality, we take $A=\mu_0=\mu_1=\kappa_0=1$ in the present paper.

There are a lot of important progress on the analysis of the global existence and dynamical behaviors of solutions for fluid-particle systems [5,10,11,13,14,17–20,22,25,27–31,36]. Among them, for incompressible NS-VFP equations, He [20] and Goudon *et al.* [17] proved the global regularity and exponential decay rate of classical solutions in spatial periodic domain, and Chae *et al.* [10] showed the global existence of weak solutions in spatial whole space. For compressible NS-VFP system with constant drag force coefficient, the global existence of weak solutions to three-dimensional initial boundary value problem with the adiabatic constant $\gamma>\frac{3}{2}$ was obtained by Mellet and Vasseur [36], the global well-posedness of strong solutions to Cauchy problem was established either for two-dimensional large initial data in [22] and for three-dimensional small initial data