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Unified Solution of Conjugate Fluid and Solid Heat Transfer–Part I. Solid Heat Conduction

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Abstract. A unified solution framework is proposed for efficiently solving conjugate fluid and solid heat transfer problems. The unified solution is solely governed by the compressible Navier-Stokes (N-S) equations in both fluid and solid domains. Such method not only provides the computational capability for solid heat transfer simulations with existing successful N-S flow solvers, but also can relax time-stepping restrictions often imposed by the interface conditions for conjugate fluid and solid heat transfer. This paper serves as Part I of the proposed unified solution framework and addresses the handling of solid heat conduction with the nondimensional N-S equations. Specially, a parallel, adaptive high-order discontinuous Galerkin unified solver has been developed and applied to solve solid heat transfer problems under various boundary conditions.

AMS subject classifications: 65M22, 76N06

Key words: Conjugate heat transfer, solid heat conduction, compressible Navier-Stokes, exponential time integration, discontinuous Galerkin.

1 Introduction

Conjugate fluid and solid heat transfer problems exist in many areas of science and engineering, such as turbomachinery, heat exchangers, and semiconductor devices. A conventional way of solving the conjugate heat transfer problem is to combine the Navier-Stokes (N-S) equations for the fluid with the Fourier-Biot (F-B) equation for the solid [6, 19], and two stand-alone solvers representing each physic are loosely coupled by exchanging physical parameters through the domain interface conditions [22,24,25]. However, coupling in this way often leads to stability constraints and very restrictive time steps [4, 20], which makes the approach less efficient. The other approach is to develop

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a fully coupled discretization method modeling both the solid and the fluid with appropriate interface conditions. However, the production of a single code with the strong coupling of the N-S and the F-B equations can be as much work as writing individual codes for separate fluid and solid applications, and is hard to utilize advanced numerical capabilities of the existing fluid and solid solvers [8,17].

For conjugate heat transfer with incompressible flows, the solid heat transfer phenomena can be modeled either by the F-B equation or using the incompressible N-S equations also in the solid domain. The latter strategy is possible since the energy equation in the incompressible N-S equations decouples from the continuity and momentum equations. However, the situation is essentially different for the compressible N-S equations where the energy equation does not decouple from the continuity and momentum equations. In [18], Nordström et al. present the similarity condition for the compressible N-S equations and the F-B equation. In their work, the velocities are uncoupled and the fluid and the solid domains are explicitly coupled by continuity of temperature and heat fluxes, but this approach still suffers from stability issues. The goal of this work is to develop a unified solution solver which can fully inherit the advanced computational capabilities of the existing N-S flow solver, resulting in an all-variable coupled method to efficiently simulate conjugate fluid and solid heat transfer problems. In the proposed method, the unified solution in both the fluid and solid domains is solely governed by the full compressible N-S equations in a nondimensional form. It belongs to the fully coupled discretization of fluid and solid which can thus relax the time step restriction existing in the loose coupling methods. Additionally, a new capable solid solver equipped with state-of-the-art computational fluid dynamics (CFD) methods can be obtained with the proposed unified solution strategy.

This paper serves as Part I of the proposed unified solution framework and focuses on applying the compressible N-S equations to model and solve the solid heat transfer problems. Various advanced CFD methods with fast exponential integrator-based time marching [9–16] can be utilized to compute the solid solutions and provide more computational capabilities than traditional solid heat transfer solvers. The capabilities of using an adaptive high-order discontinuous Galerkin (DG) N-S solver for various solid heat transfer problems will be tested with convection and radiation effects.

The remaining parts of this paper are organized as follows. Section 2 presents the theory and equations of the unified solution for solid heat conduction, and Section 3 introduces the numerical methods used in the existing N-S flow solver. Several solid heat transfer problems are tested in Section 4 to demonstrate the capability of the proposed method. Some concluding remarks are finally given in Section 5.

2 Governing equations

The full, compressible N-S equations govern the transport processes of mass, momentum, and energy, where the energy equation is proved mathematically connected to the F-B