

An Adaptive Physics-Informed Neural Network with Two-Stage Learning Strategy to Solve Partial Differential Equations

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Abstract. Physics-Informed Neural Network (PINN) represents a new approach to solve Partial Differential Equations (PDEs). PINNs aim to solve PDEs by integrating governing equations and the initial/boundary conditions (I/BCs) into a loss function. However, the imbalance of the loss function caused by parameter settings usually makes it difficult for PINNs to converge, e.g. because they fall into local optima. In other words, the presence of balanced PDE loss, initial loss and boundary loss may be critical for the convergence. In addition, existing PINNs are not able to reveal the hidden errors caused by non-convergent boundaries and conduction errors caused by the PDE near the boundaries. Overall, these problems have made PINN-based methods of limited use on practical situations. In this paper, we propose a novel physics-informed neural network, i.e. an adaptive physics-informed neural network with a two-stage training process. Our algorithm adds spatio-temporal coefficient and PDE balance parameter to the loss function, and solve PDEs using a two-stage training process: pre-training and formal training. The pre-training step ensures the convergence of boundary loss, whereas the formal training process completes the solution of PDE by balancing various loss functions. In order to verify the performance of our method, we consider the imbalanced heat conduction and Helmholtz equations often appearing in practical situations. The Klein-Gordon equation, which is widely used to compare performance, reveals that our method is able to reduce the hidden errors. Experimental results confirm that our algorithm can effectively and accurately solve models with unbalanced loss function, hidden errors and conduction errors. The codes developed in this manuscript are publicly available at <https://github.com/callmedrcom/ATPINN>.

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1. Introduction

Modeling of partial differential equations (PDEs) is an important and widely used tool of science and engineering. In turn, methods to accurately solve PDEs are of great significance. In complex industrial systems, such as digital twin and parallel control [24,36], PDE provides relevant models to lay the foundation of the whole process. Currently, besides analytical methods, the solution of a PDE may be obtained by finite difference, finite volume, and finite element methods (FEM), as well as other numerical methods. However, in the case of repetitive simulations such as optimization, control and real-time monitoring, FEM and other numerical methods are time-consuming and laborious, because forward simulation needs repeatedly solving large systems of non-linear equations.

The emergence of deep learning techniques has completely changed several research fields, such as image and speech recognition [18], video processing [25], natural language processing [54], medical imaging [21], and fault diagnosis [45]. In recent years, deep learning has been applied in scientific computing. Sirignano *et al.* [42] proposed the deep Galerkin method for solving high-dimensional PDE. E *et al.* [9] proposed the deep Ritz method to solve the variational problem. Lyu *et al.* [29] proposed a deep mixed residual method (MIM) to solve partial differential equations with high order derivatives. Physics-Informed Neural Networks (PINNs) is a new algorithm for solving differential equations which has been recently proposed [33,34]. The training process of a PINN is done using a loss function which includes the PDE governing equation and the I/BCs of PDE. Compared to FEM, PINN is a data-driven algorithm and does not require a spatial or temporal mesh. In addition, it has a wide range of potential applications and a simple derivation process [26]. Once the network is trained, the solution is obtained in a faster way compared to previous numerical methods. At present, PINN has achieved remarkable results in the field of computational science and engineering, including computational and solid mechanics [1,10,12,35], fluid mechanics [4,13,17,41,53], high frequency partial differential equations [6], ordinary differential equations [32], fault detection [38], state-space modeling [2], biomedical science [5,8,20,51], thermodynamics [55], and design of metamaterials [7,23] among others.

The main innovation of PINN is to exploit a residual network, which calculates the value of the loss function with some constraints. This process encodes the PDE to restrain the output of the neural network. PINN uses automatic differentiation for the differential operator, while traditional methods are based on numerical differentiation. As pointed out in previous work [3], automatic differentiation is the main advantage of PINN, because the operators on the remaining networks can be effectively expressed by automatic differentiation. In the prediction step, the input includes a time step