CSIAM Trans. Appl. Math.
doi: 10.4208/csiam-am.SO-2022-0040

Vol. 4, No. 3, pp. 515-541
September 2023

An Integral Equation Method for the Scattering by Core-Shell Structures in a Layered Medium

Gang Bao[1] and Lei Zhang[2,*]

[1] *School of Mathematical Sciences, Zhejiang University, Hangzhou 310058, P.R. China.*
[2] *College of Science, Zhejiang University of Technology, Hangzhou 310023, P.R. China.*

Received 26 October 2022; Accepted 9 January 2023

Abstract. The core-shell structure design is an important subject in science and engineering, which also plays a key role in wave scattering and target reconstructions. This work aims to develop a novel boundary integral equation method for solving the acoustic scattering from a 3D core-shell structure in a two-layered lossy medium. The boundary integral equation contains continuous and weakly singular kernels. The well-posedness of the scattering problem is established by combining the integral equation, variational, and operator theory techniques. The study lays the groundwork for future numerical methods for layered obstacles and rough surfaces composite scattering and inverse scattering problems.

AMS subject classifications: 35Q60, 31B10, 45L05

Key words: Composite scattering, Helmholtz equation, existence and uniqueness, integral equation method.

1 Introduction

The core-shell structure scattering arises in a wide range of scientific fields, including sea radar target detection [6], underwater radar surveillance [3], and the design of optics devices [4,14,25]. Given an incident field, the scattering problem is to determine the scattered field from the governing differential equation, along with the boundary conditions. Recent progress has been made in the development of acoustic scattering from underwater or aerial core-shell structure vehicles. In [9], a strategy was investigated to control plasmonic resonances by core-shell geometries from analyzing the scattering response of a column of partially magnetized plasma. A great deal of studies deal with various aspects of the transient response of submerged or fluid-filled elastic shell structures with

*Corresponding author. *Email addresses:* `baog@zju.edu.cn` (G. Bao), `zhanglei@zjut.edu.cn` (L. Zhang)

simple geometrical configurations. The closed-form analytic solutions for spherical or cylindrical shells have been reported in the literature, see, e.g., [10,12,21] and the references therein. However, little is known in mathematics about the scattering problems of a general shape of three-dimensional layered obstacles in an unbounded structure, which is vital for underwater exploration and many other engineering applications. The main challenges in analysis and computation include the unboundedness and complexity of interfaces, multiple scales, and multiple scattering between targets and rough interfaces.

Our goal in this work is to develop a mathematical model and to study its well-posedness for solving composite scattering problems. More specifically, we study the acoustic scattering of core-shell structures (such as low frequency plasma-coated obstacles) in a two-layered lossy medium, where the core is with an impedance boundary and the shell is with a transparent boundary. In particular, the scattering of a point source incidence is considered for the layered obstacle, interface, and shell. The wave field is governed by the three-dimensional Helmholtz equation, which describes the propagation of acoustic waves. Throughout, we assume that the medium is lossy and inhomogeneous with smooth interfaces. The assumptions are reasonable and general. In fact, in the target recognition applications above the sea surface, it is reasonable to assume that the medium is filled with two different lossy parts with smooth interfaces (air and water). The interface can be very general, including plane structures, periodic structures (cf. [1]) and general unbounded rough surfaces (cf. [26,27]). Also, the wavenumbers in the medium may have different positive imaginary parts. Recently, related problems have been studied extensively. In [8], a fast numerical method was proposed for calculating the electromagnetic scattering from a perfectly electric conducting object above a two-layered dielectric rough surface. The mode-expansion method for calculating electromagnetic waves scattered by objects on rough ocean surfaces was considered in [28]. In [11], the method of moments was used to rigorously analyze the wide-band VHF scattering from a perfectly conducting trihedral placed above a lossy, dispersive half-space. A Kirchhoff-type formula for transient elastic waves was originally introduced by Love [19] for the fluid-solid interaction scattering problem. Helmholtz-type integral formulas were systematically derived for elastic waves in isotropic and anisotropic solids by Pao [23]. Various systems of boundary integral equations over the interface between the fluid and the solid have been derived and analyzed by Luke and Martin [20]. Hsiao *et al.* [13] presented weak formulations of the fluid-solid interaction problem by coupling the field and boundary integral equation methods. The relevant functional analysis, which is necessary for the theory's treatment and the numerical solution of linear integral equations, can be found in [16,22]. More recently, a boundary integral equation has been proposed for solving 2D homogeneous obstacle acoustic (electromagnetic) composite scattering problems. Based on the energy estimates, the uniqueness of the solution or the scattering problem is established [2,18]. To our best knowledge, no mathematical study is available for the scattering of the core-shell structures in a layered medium.

In this paper, we derive a rigorous mathematical model for a class of scattering problems from 3D core-shell structures in a two-layered lossy medium with an unbounded