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EFFICIENT NONNEGATIVE MATRIX FACTORIZATION VIA MODIFIED MONOTONE BARZILAI-BORWEIN METHOD WITH ADAPTIVE STEP SIZES STRATEGY*

Wenbo Li
Department of Applied Mathematics, Xi'an University of Technology, Xi'an 710054, China;
College of Mathematics and Statistics, Xi'an Jiaotong University, Xi'an 710049, China
Email: wbli57@hotmail.com
Jicheng Li[1)] and Xuenian Liu
College of Mathematics and Statistics, Xi'an Jiaotong University, Xi'an 710049, China
Email: jcli@mail.xjtu.edu.cn, lxn901018@163.com

Abstract

In this paper, we develop an active set identification technique. By means of the active set technique, we present an active set adaptive monotone projected Barzilai-Borwein method (ASAMPBB) for solving nonnegative matrix factorization (NMF) based on the alternating nonnegative least squares framework, in which the Barzilai-Borwein (BB) step sizes can be adaptively picked to get meaningful convergence rate improvements. To get optimal step size, we take into account of the curvature information. In addition, the larger step size technique is exploited to accelerate convergence of the proposed method. The global convergence of the proposed method is analysed under mild assumption. Finally, the results of the numerical experiments on both synthetic and real-world datasets show that the proposed method is effective.

Mathematics subject classification: 15A23, 65F30.
Key words: Adaptive step sizes, Alternating nonnegative least squares, Monotone projected Barzilai-Borwein method, Active set strategy, Larger step size.

1. Introduction

For a given nonnegative data matrix $V \in \mathbb{R}^{m\times n}$ and a pre-specified positive integer $r < \min(m,n)$, we consider in this paper the following optimization problem

$$\min_{W\in\mathbb{R}^{m\times r}, H\in\mathbb{R}^{r\times n}} f(W,H) \equiv \frac{1}{2}\|V-WH\|_F^2, \quad \text{subject to } W\geq 0, H\geq 0, \tag{1.1}$$

where $\|\cdot\|_F$ is the Frobenius norm. This problem is known as nonnegative matrix factorization (NMF) and has been used in many fields such as image processing [8], text mining [18], machine learning [15], etc.

In the last decade, many efficient methods have been proposed to solve (1.1). The most well-known and representative method is the multiplicative update (MU) algorithm [13, 14] which was first proposed by Lee and Seung in 1999. The MU algorithm updates the two matrices by using the gradient descent method at every step, however, the MU algorithm may not converge to a stationary point (see [20]).

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[1)] Corresponding author

Another commonly used approach is the alternating nonnegative least squares (ANLS) framework which is to optimize W and H by alternately solving the following nonnegative least squares subproblems:

$$W^{k+1} = \min_{W\geq 0} f(W, H^k), \tag{1.2}$$

$$H^{k+1} = \min_{H\geq 0} f(W^{k+1}, H). \tag{1.3}$$

In [6], Grippo and Sciandrone have shown that any limit point of the sequence generated by ANLS is a stationary point of (1.1). Since the role of W and H is perfectly symmetric for the problem (1.1), therefore, many approaches only focus on solving (1.2), which include quasi Newton method [12], alternating projected Barzilai-Borwein methods [9], projected gradient method [16]. However, these methods may be inefficient due to the time-consuming line search.

Recently, a new researching [7] has shown that applying Nesterov's optimal gradient method (OGM) to solve the subproblem (1.2) or (1.3) without line search can converge faster. However, Huang et al. [10] found that OGM might take lots of iterations to reach a given tolerance which might degrade the efficiency of NeNMF [7]. In [10], the authors presented a quadratic regularization projected Barzilai-Borwein (QRPBB) method, and showed that the QRPBB method improves the performance of the projected Barzilai-Borwein method significantly and outperforms other three methods including PG [16], APBB2 [9], and NeNMF [7]. However, the QRPBB method is time-consuming for checking the nonmonotone line search. To overcome the drawback, a monotone projected Barzilai-Borwein (MPBB) [11] method was suggested to solve (1.2) or (1.3), in which the step size is determined without line search. But, by the analysis of the MPBB method, we found that it still spent a lot of iterations to reach a given tolerance, which is a serious disadvantage for large-scale problems. It motivates us to develop much faster algorithm for solving the subproblem (1.2) or (1.3).

In this paper, we first modify the active set identifying technique in [19], and then by using the modified active set identifying technique, we propose an efficient method to solve the subproblem (1.2) or (1.3) based on the MPBB method in [11], in which by using the norm of gradient we design a adaptive step sizes selection strategy so that the Barzilai-Borwein (BB) step sizes [1, 2] can be adaptively selected to improve the convergence rate. The numerical experiments show that our adaptive step sizes strategy is superior to the alternate step sizes strategy [3] in some cases. At each iteration, the ASAMPBB method adopts the modified identification technique to eastimate the active and free variables, and the adaptive monotone projected Barzilai-Borwein gradient method is used in the free subspace. Moreover, our ASAMPBB method exploits the larger step size technique to accelerate convergence. Unlike the APBB4 [9] and MPBB [11] methods, our ASAMPBB method uses the curvature information to obtain the optimal step size. The global convergence result is established under mild conditions. Numerical experiments on synthetic and real-world datasets indicate the proposed method is encouraging.

The manuscript is organized as follows: First, in Section 2, we propose an efficient algorithm and establish its global convergence. The experimental results are presented in Section 3, where synthetic and real-world datasets are used to demonstrate the performance of the proposed algorithm. Finally, Section 4 concludes the manuscript. Throughout the paper, the symbol $\|.\|_F$ denotes the Frobenius norm of matrixes, $\langle ., . \rangle$ denotes the inner product of two matrixes.