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Numerical Simulations of the Richtmyer–Meshkov Instability of Solid-Vacuum Interface

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Abstract. The Richtmyer–Meshkov instability of interfaces separating elastic-plastic materials from vacuum is investigated by numerical simulation using a multi-material solid mechanics algorithm based on an Eulerian framework. The research efforts are directed to reveal the influence of the initial perturbation and material strength on the deformation of the perturbed interface impacted by an initial shock. By varying the initial amplitude ($k\xi_0$) of the perturbed interface and the yield stress (σ_Y), three typical modes of interface deformation have been identified as the broken mode, the stable mode and the oscillating mode. For the broken mode, the interface width (i.e., the bubble position with respect to that of the spike) increases continuously resulting in a final separation of the spike from the perturbed interface. For the stable mode, the interface width grows to saturation and then maintains a nearly constant value in the long term. For the oscillating mode, the wavy-like interface moving forward obtains an aperiodic oscillation of small amplitude, namely, the interface width varies in time slightly around zero. The intriguing difference of the typical modes is interpreted qualitatively by comparing the early-stage wave motion and the commensurate pressure and effective stress. Further, the subsequent interface deformation is illustrated quantitatively via the time series of the interface positions and velocities of these three typical modes.

AMS subject classifications: 74C15, 76L05, 76E17

Key words: Richtmyer–Meshkov instability, elastic-plastic flow, interface deformation mode.

1 Introduction

The Richtmyer–Meshkov instability (RMI) named after the seminal work of Richtmyer [1] and Meshkov [2] describes the growing behavior of a perturbed interface between two

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materials of different properties when a shock wave passes through. The misalignment between the local density gradient across the interface and the shock pressure gradient, which are perpendicular to the interface and the shock wave, respectively, causes the baroclinic vorticity generation near the interface. The perturbed interface grows unboundedly unless there exists a mechanism that could dissipate or advect such vorticity. This problem was originally regarded as the impulsive limit of the Rayleigh–Taylor instability (RTI) [3,4], in which the perturbed interface between two materials is under the effect of a continuous acceleration. However, in recent years, the RMI has become increasingly important because it controls a variety of processes in nature and technology such as inertial confinement fusion (ICF) [5] and astrophysical problems [6]. The study of RMI is of fundamental importance to improve ICF experiment equipment and to gain a better understanding of the core-collapse supernova mechanism.

The RMI has been widely studied in the fluid dynamics in the past 60 years, including numerical simulation, experimental research and theoretical analysis [7–9]. Specifically, it has been extensively investigated from the fluid dynamics perspective and appears in problems of interest in solid mechanics to make the simulation parameters approximate to the ICF experiment. In solid mechanics, researches of purely elastic RMI flow are studied based on the analytical model [10] and linearized analysis [11] and the results show that the interface is always stable because the vorticity deposited on the material interface during shock passage is propagated away by the shear waves. Since the plasticity theory is nonlinear and thus complicated, the study of the RMI in elastic-plastic material has to mainly rely on numerical approaches other than purely analytical models.

The first attempt to describe the growing behavior of elastic-plastic RMI was conducted by Piriz et al. [12], who developed an analytical model for the RMI in solids that is constructed by assuming that the shocked material behaves similar to an elastic-perfectly plastic material described by the Prandtl–Reuss flow rule with the von Mises yield criterion. They performed extensive two-dimensional numerical simulations with the finite element code ABAQUS for comparison with the analytical results and they found an excellent agreement between the model and simulations. They reported that the interface oscillates around an average amplitude in the long term and the relationship for the long-term average amplitude of the interface can be calculated by a function of the material parameters and initial conditions,

$$\xi_m - \xi_0 = C\frac{\rho_0\dot{\xi}_0^2}{k\sigma_Y}, \tag{1.1}$$

where ξ_m and ξ_0 are the long-term and initial interface amplitudes, respectively, $\dot{\xi}_0 = k\xi_0 u_p$ is the initial interface growth rate, u_p is the particle velocity after the shock, ρ_0 is the unstressed density of material, k is the wave number of the initial perturbation, σ_Y is the yield stress of the material, the coefficient $C = 0.29$ is obtained by numerical simulation fitting.

Dimonte et al. [13] later proposed a model for the RMI at a metal-gas interface to infer the metal's yield stress (σ_Y) under shock loading and release. The RMI inferred yield