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A Node-Based Smoothed Finite Element Method with Linear Gradient Fields for Elastic Obstacle Scattering Problems

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Abstract. In this paper, a node-based smoothed finite element method (NS-FEM) with linear gradient fields (NS-FEM-L) is presented to solve elastic wave scattering by a rigid obstacle. By using Helmholtz decomposition, the problem is transformed into a boundary value problem with coupled boundary conditions. In numerical analysis, the perfectly matched layer (PML) and transparent boundary condition (TBC) are introduced to truncate the unbounded domain. Then, a linear gradient is constructed in a node-based smoothing domain (N-SD) by using a complete order of polynomial. The unknown coefficients of the smoothed linear gradient function can be solved by three linearly independent weight functions. Further, based on the weakened weak formulation, a system of linear equation with the smoothed gradient is established for NS-FEM-L with PML or TBC. Some numerical examples also demonstrate that the presented method possesses more stability and high accuracy. It turns out that the modified gradient makes the NS-FEM-L-PML and NS-FEM-L-TBC possess an ideal stiffness matrix, which effectively overcomes the instability of original NS-FEM. Moreover, the convergence rates of L^2 and H^1 semi-norm errors for the two NS-FEM-L models are also higher.

AMS subject classifications: 35L05, 65N99

Key words: Elastic obstacle scattering, Helmholtz equations, perfectly matched layer, transparent boundary condition, NS-FEM with linear gradient.

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1 Introduction

The obstacle scattering [1] has been widely used in medicine, location detection and other fields. It can be divided into acoustic, electromagnetic and elastic scattering. These problems have been widely studied in theoretical [2–4] and numerical [5,6] aspects.

A fundamental difficulty in the obstacle scattering is that the problem domain is open. Hence some techniques need to be applied to make the problem domain truncated. At present, there are many techniques have been studied, among which the common ones include perfectly matched layer (PML) [7], transparent boundary condition (TBC) [3]. The PML refers to the method of applying a layer with a special absorption medium layer to a certain domain around the obstacle, so that the wave can be fully absorbed upon reaching the outer boundary of PML. Many researchers have proved that PML is a valid method and is widely used in solving acoustic wave [8–11], elastic wave [12–16] and electromagnetic wave scattering [4,7]. The TBC is also a common technique, which is constructed using the analytic solutions with an infinite Fourier series. By imposing the TBC on the boundary of the truncated domain, the reflection of the wave can be avoided. The TBC has been used for solving many wave scattering problems [17–24].

Both compressional wave and shear wave exist in the scattering of elastic waves, which makes the study of elastic waves more complicated and it is not easy to obtain analytical solutions for arbitrarily shaped obstacles. Currently, many discrete numerical methods are proposed for solving these problems, such as the boundary integral method [1], finite element method (FEM) [25,26], smoothed point interpolation method (S-PIM) [27], and smoothed finite element method (S-FEM) [28]. Since PML and TBC are artificial boundary conditions in nature, there will be certain errors when they are applied. Usually, due to the over-stiff property of FEM, the solution accuracy of FEM model with the TBC or PML is not very high for solving this problem. In order to make up for this deficiency of FEM, Liu et al. proposed the G space theory based on weakened weak (W2) formulations and constructed S-FEM models [29,30]. Besides, according to different type of smoothing domains, the S-FEM can be divided into the cell-based S-FEM (CS-FEM), the node-based S-FEM (NS-FEM), the edge-based S-FEM (ES-FEM) in 2D problem. These models can obtain high precision solutions for different problems, such as solid mechanics problems [31–33], thermal problems [34] and so on. Recently, Yue and Wu proposed ES-FEM model with PML technique [5] and TBC technique [35] for solving elastic wave obstacle scattering, respectively. The NS-FEM has been proved to have many properties for solid mechanics, such as, spatial discrete stability, time response stability and possessing near-accurate stiffness and so on [36,37]. But we noticed that the original NS-FEM cannot be extended to wave scattering problems due to the "over-soft" stiffness of the method. Chai first presented a stable NS-FEM (SNS-FEM) for the analysis of acoustic scattering to cure the instability of original NS-FEM [23], and Wang solved the elastic wave obstacle scattering problem by using SNS-FEM and PML technique [38]. In addition, Liu proposed a novel pick-out technique for constructing higher order smoothed derivatives [39]. Li and Liu also extended this technique to NS-FEM (NS-FEM-L), which