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A Diffusive Predator-Prey Model with Spatially Heterogeneous Carrying Capacity

CHEN Jiawei and WANG Biao*

College of Science, Xi'an University of Science and Technology, Xi'an 710054, China.

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Abstract. We study local dynamics of a diffusive predator-prey model in a spatially heterogeneous environment, where intrinsic growth rate of the prey is spatially homogeneous, whereas carrying capacity of the habitat is spatially inhomogeneous. In comparison with the existing predator-prey models, the stability of semi-trivial steady state of this model displays distinct properties. For example, for certain intermediate ranges of the death rate of the predator, the semi-trivial steady state can change its stability at least once as the dispersal rate of the prey varies from small to large, while the stability of the semi-trivial steady state is immune from the dispersal rate of the predator.

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1 Introduction

The movement of organisms is often crucial to its persistence. The creatures are scattered to look for resources, seek breeding habitat, and avoid predation, etc. Understanding the impact of dispersal on population dynamics is still an important topic in ecology. One way to investigate how the joint action of dispersal and spatial heterogeneity influences populations and communities is by using reaction-diffusion models [1]. For instance, it was shown in [2] that for a reaction-diffusion model with logistic growth term in spatially heterogeneous environments, as long as a species keeps moving randomly, the total amount of resources always supports a population strictly larger than the total carrying capacity. Recently, this model has been generalized to be a more realistic one [3], where

*Corresponding author. *Email addresses:* `20201103003@stu.xust.edu.cn` (J. W. Chen), `wang.biao@xust.edu.cn` (B. Wang)

both intrinsic growth rate and carrying capacity depend on spatial variable in the habitat. Their outcomes indicate that the total population of the species has more complicated relations with the total carrying capacity. However, when the intrinsic growth rate is constant, while the carrying capacity is spatially heterogeneous, a striking result from [4] implies that for any diffusion rate, the total amount of resources supports a population strictly smaller than the total carrying capacity. For more research concerning the effects of diffusion rate and spatial heterogeneity of the environment on dynamics of populations via reaction-diffusion models, we refer interested readers to [5–13] and reference therein.

In this paper, we discuss a diffusive predator-prey model with spatially homogeneous intrinsic growth rate of the prey and spatially inhomogeneous carrying capacity of the habitat, and explore the effects of dispersal and spatial heterogeneity on the local dynamics of the predator and prey populations. The mathematical model can be characterized by the following reaction-diffusion system:

$$\begin{cases} u_t = \mu\Delta u + u\left(1-\dfrac{u}{K(x)}\right) - uv & \text{in } \Omega\times(0,\infty), \\ v_t = \nu\Delta v + (ku-d)v & \text{in } \Omega\times(0,\infty), \\ \dfrac{\partial u}{\partial n} = \dfrac{\partial v}{\partial n} = 0 & \text{on } \partial\Omega\times(0,\infty), \\ u(x,0)=u_0(x),\ v(x,0)=v_0(x) & \text{in } \Omega, \end{cases} \tag{1.1}$$

where $u(x,t)$ and $v(x,t)$ represent the population density of prey and predator species at location x and time t with corresponding diffusion rates μ and ν. The initial values $u_0(x)$ and $v_0(x)$ are both non-negative and non-trivial. The function $K(x)$ denotes carrying capacity of the habitat, and $d>0$ is the mortality rate of the predator. $\Delta := \sum_{i=1}^{N} \partial^2/\partial x_i^2$ is the Laplace operator in $\mathbb{R}^N$, which characterizes the random movement of the predator and the prey species. The habitat Ω is a bounded domain in $\mathbb{R}^N$ with smooth boundary $\partial\Omega$. The zero Neumann boundary conditions mean that no individual can cross the boundary of the habitat. $\partial u/\partial n = \nabla u\cdot n$, where n is the outward unit normal vector on $\partial\Omega$. The constants μ,ν and k are supposed to be positive.

To reflect spatial heterogeneity of carrying capacity of the habitat in (1.1), throughout this paper, we always assume that the carrying capacity $K(x)$ satisfies the following condition:

$$K(x)>0, \text{ is non-constant, and Hölder continuous on } \bar{\Omega}. \tag{1.2}$$

If $K(x)$ satisfies (1.2), then the single species equation [3,4]

$$\begin{cases} \mu\Delta u + u\left(1-\dfrac{u}{K(x)}\right) = 0 & \text{in } \Omega, \\ \dfrac{\partial u}{\partial n} = 0 & \text{on } \partial\Omega \end{cases} \tag{1.3}$$