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A Note on Asymptotic Stability of Rarefaction Wave of the Impermeable Problem for Radiative Euler Flows

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Abstract. This paper is devoted to studying the initial-boundary value problem for the radiative full Euler equations, which are a fundamental system in the radiative hydrodynamics with many practical applications in astrophysical and nuclear phenomena, with the slip boundary condition on an impermeable wall. Different from our recent paper named "Asymptotic stability of rarefaction wave with slip boundary condition for radiative Euler flow", in this paper we study the initial-boundary value problem with the Neumann boundary condition instead of the Dirichlet boundary on the temperature. Based on the Neumann boundary condition on the temperature, we obtain that the pressure also satisfies the Neumann boundary condition. This observation allows us to establish the local existence and a priori estimates more easily than the case of the Dirichlet boundary condition which is studied in the mentioned paper. Since for the impermeable problem, there are quite a few results available for the Navier-Stokes equations and the radiative Euler equations, it will contribute a lot to our systematical study on the asymptotic behaviors of the rarefaction wave with the radiative effect and different boundary conditions such as the inflow/outflow problem and the impermeable boundary problem in our series papers.

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1 Introduction

The radiative full Euler equations are a fundamental system to describe the motion of the compressible gas with the radiative heat transfer phenomena, which has many applications in astrophysics and nuclear explosions. Mathematically, the one-dimensional radiative full Euler equations in the Eulerian coordinates are a hyperbolic-elliptic coupled system of the following form:

$$
\begin{cases}
\rho_t+(\rho u)_x=0, & \text{(1.1a)}\\
(\rho u)_t+(\rho u^2+p)_x=0, & \text{(1.1b)}\\
\left\{\rho\left(e+\dfrac{u^2}{2}\right)\right\}_t+\left\{\rho u\left(e+\dfrac{u^2}{2}\right)+pu\right\}_x+q_x=0, & \text{(1.1c)}\\
-q_{xx}+aq+b(\theta^4)_x=0, & \text{(1.1d)}
\end{cases}
$$

where ρ,u,p,e and θ are respectively the density, velocity, pressure, internal energy and absolute temperature of the gas, and q is the radiative heat flux. Positive constants a and b depend only on the gas itself. Like the classic compressible Euler equations, the Eqs. (1.1a)-(1.1c) stand for the conservation of the mass, momentum and energy respectively. The Eq. (1.1d) is related to the radiative heat transfer phenomenon, and one can refer [1,12,23,29,36,40] for more details. System (1.1) can also be derived by the non-relativistic limit (speed of light tending to $+\infty$) from a hyperbolic-kinetic system, and rigorous mathematical derivation can be found in [16]. Throughout this paper, we will concentrate on the ideal polytropic gas

$$
p=R\rho\theta,\quad e=C_v\theta,\quad C_v=\frac{R}{\gamma-1}, \tag{1.2}
$$

where $\gamma>1$ is the adiabatic exponent and $R>0$ is the specific gas constant.

In this paper, we will investigate the initial-boundary value problem of system (1.1) on $0\le x<+\infty$ and $0\le t<+\infty$ with the initial data

$$
(\rho,u,\theta)(x,0)=(\rho_0,u_0,\theta_0)(x)\quad \text{for}\quad x\ge 0,\quad \text{and}\quad \inf_{x\in\mathbb{R}^+}(\rho_0,\theta_0)(x)>0, \tag{1.3}
$$