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A NEWTON-TYPE GLOBALLY CONVERGENT INTERIOR-POINT METHOD TO SOLVE MULTI-OBJECTIVE OPTIMIZATION PROBLEMS*

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Abstract

This paper proposes an interior-point technique for detecting the nondominated points of multi-objective optimization problems using the direction-based cone method. Cone method decomposes the multi-objective optimization problems into a set of single-objective optimization problems. For this set of problems, parametric perturbed KKT conditions are derived. Subsequently, an interior point technique is developed to solve the parametric perturbed KKT conditions. A differentiable merit function is also proposed whose stationary point satisfies the KKT conditions. Under some mild assumptions, the proposed algorithm is shown to be globally convergent. Numerical results of unconstrained and constrained multi-objective optimization test problems are presented. Also, three performance metrics (modified generational distance, hypervolume, inverted generational distance) are used on some test problems to investigate the efficiency of the proposed algorithm. We also compare the results of the proposed algorithm with the results of some other existing popular methods.

Mathematics subject classification: 65N06, 65B99.
Key words: Cone method, Interior point method, Merit function, Newton method, Global convergence.

1. Introduction

The vast majority of practical optimization problems [2,16,31,33] consist of multi-objective problems. Applications of multi-objective optimization problems can be found in a various fields, including engineering design [2], optimal control systems [31], chemical engineering [33], machine learning [16], etc. Therefore, identification and characterization of the solutions to MOPs have become a very important task. MOPs consider to optimize several conflicting objectives simultaneously. Therefore, most often, a single solution that performs well for each objective functions does not exist. In solving MOP problems, sometimes decision makers come up with a compromise solution by analyzing a set of points that are representative of the entire Pareto set [29]. A feasible point is called Pareto optimal (non-dominated point) if no objective

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can be improved without sacrificing at least one other objective. When solving an MOP, the goal is to identify all possible Pareto optimal solutions.

MOPs have been solved through several techniques [23] over the last few years. The reputed classical methods such as weighted sum [19, 24], ϵ-constraint [18], physical programming [26], normal boundary intersection [5], etc., are known to find the Pareto optimal solutions. However, these methods either not able to yield a complete Pareto front or require some prior information regarding its location. Recently, a cone method [12] has been established that can generate all Pareto solutions, and no knowledge about the position of the Pareto front is required.

The formulation of cone method is found to be similar to the Pascoletti-Serafini [30] technique for vector optimization. Cone method [12] has the ability to generate both convex and nonconvex parts of the Pareto front. The formulation of the cone method (see Section 2) transforms the multi-objective optimization problem into a set of direction-based-parametric single objective problem. Although the formulation of cone method is detailed in [12], how to effectively solve the formulated direction-based parametric subproblems is not given therein. In this paper, we concern towards this direction and attempt to apply interior-point method to solve the subproblems.

In 1955, foundation of the interior-point approach was laid introduced by Frisk [11]. Subsequently, Fiacco and McCormick [10] reformulate the problem $\min\{f(x) : c(x) = 0, x \geq 0, c(x) \in \mathbb{R}^m, x \in \mathbb{R}^n\}$ as an unconstrained minimization problem and proved the global convergence of interior-point method. In 1960's, one type of interior point methods (classical log-barrier method) was used broadly. In 1970's, it was proven [27] that the Hessians for barrier methods are ill-conditioned near the optima. Therefore, despite fair finding of the log-barrier method, other methods became primary topics for research.

In 1984, Karmarkar [17] published an algorithm that solves linear programming problems in polynomial time. This was a huge improvement over existing simplex method, which solved linear programming problems in the worst-case by exponential time. In a very short time, it was found that Karmarkar's algorithm was equivalent to the log-barrier method [13]. Thereafter, the interest in interior-point methods resurged.

In 1989, Megiddo [25] first presented an interior-point method, which simultaneously solves the primal and dual problems and describes the properties of primal-dual central path for linear programming. Thereafter, a primal-dual barrier method to solve linear programming problems was implemented in [20]. To solve linear programming problems and quadratic programming problems, widely used method was barrier methods [32, 41].

As a result of the popularity of interior-point methods for linear and quadratic programming studies on their use for nonlinear optimization continue till today. The proposed method exploits the efficiencies of the cone method [12] and interior-point method. In this work, we introduce a novel differentiable merit function that helps to decide the convergence of the proposed algorithm towards the solution. The stationary points of this merit function satisfy the perturbed KKT conditions. Further, a Newton-type method is applied to solve KKT conditions. We also present the global convergence results of the proposed method.

This paper is structured as follows. In Section 2, we provide the required terminologies and notations, and briefly explain the cone method. In Section 3, we formulate an interior-point method for a nonlinear problem, which is formulated in Section 2, and find the search direction formulas. In Section 4, a merit function and its properties are presented. In Section 5, we show that the proposed algorithm is globally convergent. Section 5, refers to numerical results of the proposed method. Finally, Section 6 ends with a few concluding remarks.