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A High-Order Localized Artificial Diffusivity Scheme for Discontinuity Capturing on 1D Drift-Flux Models for Gas-Liquid Flows

Adyllyson H. Nascimento and Eugênio S. Rosa*

Department of Energy, School of Mechanical Engineering, University of Campinas, Campinas, Brazil

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Abstract. A computational code is developed for the numerical solution of one-dimensional transient gas-liquid flows using drift-flux models, in isothermal and also with phase change situations. For these two-phase models, classical upwind schemes such as Roe- and Godunov-type schemes are generally difficult to derive and expensive to use, since there are no treatable analytic expressions for the Jacobian matrix, eigenvalues and eigenvectors of the system of equations. On the other hand, the high-order compact finite difference scheme becomes an attractive alternative on these occasions, as it does not make use of any wave propagation information from the system of equations. The present paper extends the localized artificial diffusivity method for high-order compact finite difference schemes to solve two-phase flows with discontinuities. The numerical method has simple formulation, straightforward implementation, low computational cost and, most importantly, high-accuracy. The numerical methodology proposed is validated by solving several numerical examples given in the literature. The simulations are sixth-order accurate and it is shown that the proposed numerical method provides accurate approximations of shock waves and contact discontinuities. This is an essential property for simulations of realistic mass transport problems relevant to operations in the petroleum industry.

AMS subject classifications: 35L65, 65M06, 76L05, 76M20, 76N15

Key words: Compressible two-phase flows, drift-flux model, localized artificial diffusivity, high-order numerical methods.

1 Introduction

The investigation of gas-liquid flows has become increasingly important in engineering design and applications in petroleum, chemical, geothermal and nuclear industries.

*Corresponding author.
Emails: adyllyson@gmail.com (A. Nascimento), erosa@fem.unicamp.br (E. Rosa)

Mathematical modeling under steady-state and transient conditions is of great importance for flow prediction in terms of pressure, temperature, phase velocities and phase holdups. It is possible to predict steady-state flow along hilly terrains, as well as in transient flows for shut-off or re-start the line. The design of a new line or the shut-off or re-start lines requires the numerical solution of the mass, momentum and energy conservation equations.

Due to the complexity of gas-liquid flows, different two-phase models have been proposed in the literature to model the phenomena inherent to such flows [1,39,46]. A category of models that has been widely used to simulate gas-liquid flows is the drift-flux two-phase models. Several works in the literature use the drift-flux two-phase models with the hypothesis of isothermal flow, so the model has three equations, being a mass equation for each phase and a momentum equation for the gas-liquid mixture [32,38,56]. The latter results from the sum of the momentum equations for each phase, implying the elimination of complex modeling interfacial terms. However, this introduces the need for a closure law regarding the slip between the phases [57]. An advantage of the drift-flux model is that the system of equation is intrinsically stable.

The complexity of the closure laws severely restricts the possibilities of constructing numerical schemes that explicitly incorporate the physics of wave propagation into their formulations [17,19,44]. This is due to the difficulties in obtaining an analytical expression of the Jacobian matrix through purely algebraic manipulations. Nevertheless, some numerical schemes have been proposed for the isothermal drift-flux model. Romate [44] presented an approximate Riemann solver of Roe using a fully numerical approach. Evje and Fjelde [18] proposed an AUSM scheme that is not based on algebraic manipulation of the Jacobian, but makes explicit use of the approximate eigenvalues associated with the non-linear waves. Flåtten and Munkejord [22] derived a Roe-type Riemann solver with a linearized form of the Jacobian matrix obtained analytically. However, the eigenvalues were evaluated numerically. Santim and Rosa [45] also proposed a Roe-type Riemann solver, presenting an approximate analytical form for both the Jacobian matrix and the eigenvalues of the system. It is worth mentioning that all the numerical schemes mentioned above are at best second-order accurate.

To model the interfacial mass transfer between the phases it is necessary to add the energy conservation equation to the two-phase models. These models differ mainly in the level of disequilibrium between the two phases that they are able to take into account. The most complete model has seven equations [3], with an equation for the balances of mass, momentum and energy for each phase, in addition to an advection equation for the volume fraction. This model allows for the disequilibrium of pressure, temperature, velocity and Gibbs free energy between phases. Adding some equilibrium hypothesis reduces the number of equations needed in the two-phase models. The simplest model is the homogeneous equilibrium model (HEM) [9], which assumes that the phases have the same velocity and are in fully thermodynamic equilibrium. This model has three equations referring to the gas-liquid mixture, being the mass, momentum and energy conservation equations. One level of hierarchy above the HEM model is the homoge-