

# Multi-Breather, Rogue Wave and Multi-Bright-Dark Soliton Interaction of the (2+1)-Dimensional Nonlocal Fokas System

Xue-Wei Yan<sup>1</sup>, Yong Chen<sup>1,\*</sup>, Shou-Fu Tian<sup>2</sup> and Xiu-Bin Wang<sup>2</sup>

<sup>1</sup>*School of Mathematics, Harbin Institute of Technology, Harbin 150001, P.R. China.*

<sup>2</sup>*School of Mathematics, China University of Mining and Technology, Xuzhou 221116, P.R. China.*

*Received 19 September 2022; Accepted (in revised version) 30 January 2023.*

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**Abstract.** We study the (2+1)-dimensional nonlocal Fokas system by using the Hirota's bilinear method. Firstly, a general tau-function of Kadomtsev-Petviashvili (KP) hierarchy satisfied with the bilinear equation under nonzero boundary condition is derived by considering differential relations and a variable transformation. Secondly, two Gram-type solutions are utilized to the construction of multi-breather, high-order rogue wave, and multi-bright-dark soliton solutions. Then the corresponding parameter restrictions of these solutions are given to satisfy with the complex conjugation symmetry. Furthermore, we find that if the parameter  $p_{il}$  takes different values, the rogue wave solution can be classified as three types of states, such as dark-dark, four-peak and bright-bright high-order rogue wave. If the parameter  $c_i$  takes different values, the soliton solution can be classified as three type of states, including the multi-dark, multi-bright-dark and multi-bright solitons. By considering third-type of reduced tau-function to the Hirota's bilinear equations, we give the collisions between the high-order rogue wave and the multi-bright-dark solitons on constant ( $N$  is positive even) or periodic background ( $N$  is positive odd). In order to understand the dynamics behaviors of the obtained solutions better, the various rich patterns are theoretically and graphically analyzed in detail.

**AMS subject classifications:** 65M10, 78A48

**Key words:** (2+1)-dimensional nonlocal Fokas system, KP hierarchy reduction, multi-breather wave, high-order rogue wave, multi-bright-dark soliton.

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## 1. Introduction

The parity ( $\mathcal{P}$ ) and the time ( $\mathcal{T}$ ) symmetries proposed by Bender and Boettcher [5] as the one type of important discrete symmetries were used to replace the Hermiticity of the Hamiltonians in quantum theory. In classic quantum theory, Hermiticity guarantees

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\*Corresponding author. *Email addresses:* xwyan16@163.com (X.Y. Yan), yongchen@hit.edu.cn (Y. Chen)