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# Numerical Study of Fluid-Solid Interaction in Rotational Extrusion Flow

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**Abstract.** The fluid particle dynamics method is employed to study the fluid-solid interaction problem, which can avoid the explicit implementation of fluid particle boundary condition and capture the hydrodynamic interaction well. We solve the incompressible Navier-Stokes equation coupled with the rigid body motion equation in polar or cylindrical coordinates. A pressure stabilization scheme is used to solve the system in polar coordinates for two dimensional case and cylindrical coordinates for three dimensional case. Our objective is to understand numerically the fluid-solid interaction in rotational extrusion flow. We numerically verify the correctness of method presented here and give comparative analysis for different parameters. We present the Jeffery orbit formulation in annular region. Numerical experiments show that the fluid particle dynamics method is reliable and efficient for numerical simulation of particulate flow in cylindrical coordinate system.

**AMS subject classifications**: 76-10

**Key words**: Fluid particle dynamics method, cylindrical coordinates, incompressible Navier-Stokes, finite difference, repulsive force.

## 1. Introduction

The coupling of fluid-solid mechanics is a branch of mechanics generated by the intersection of fluid and solid mechanics. Its important feature is the interaction between two-phase media, including the deformation or movement of solid structure under the action of fluid load, and the influence of solid deformation or movement on flow field. It is the interaction between fluid and solid structure that produces various fluid-solid coupling phenomena under different conditions. In fact, such problems appear in various areas, including strong wind swing in high-rise buildings construction [18,21], interaction between blood and heart in bioengineering [27], complex stress and temperature experienced by

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high molecular polymers during processing [2, 17], etc. Therefore, numerical simulations of fluid-solid coupling is of great importance. However, researchers rarely focus on the problem of fluid-solid interactions when multiple particles are subjected to rotational extrusion in cylindrical instruments. In this paper, we want to explain and validate part of the work of the rotary extrusion rheometer – a machine that studies the rheological behavior of polymers – by studying the fluid-structure interaction in a hollow cylinder.

There are many numerical simulation methods for the coupling problem of incompressible viscous fluid and solid particles — e.g. arbitrary Lagrangian-Eulerian method (ALE), fictitious domain method (FDM), immersed boundary method (IBM), and fluid particle dynamics (FPD) method. The ALE method directly simulating the motion of fluid and particles is based on moving unstructured grids. It needs remeshing and projection [12, 13]. The fictitious domain method extends the actual computational domain inside or outside of the desired object to form a fictitious computational domain with a simple shape, so that a structured grid can be used to solve it efficiently [8]. The fictitious domain method has evolved different algorithms according to different problems. The distributed Lagrange multiplier method is proposed in [7]. The immersed boundary method, introduced by Peskin and McQueen [26], is widely used in fluid and solid simulations. It uses non-boundary-conforming meshes in numerical discretization, and the no-slip condition on the surface of the immersed object is enforced by adding a volume force to the momentum equation. The FPD method was proposed by Tanaka and Araki [24]. It is a diffusion interface method, which can avoid the explicit implementation of fluid-solid boundary condition and capture the hydrodynamic interaction well. This method has efficient numerical simulation performance, and has been well verified by numerical experiments [25].

In 2005, the effect of particle size on the lateral migration of particles in rectangular microchannels with different nozzle shapes at low Reynolds numbers was studied by Staben *et al.* [22]. The particle loaded flow in two-dimensional channels was studied in [3] using the direct numerical simulation method of coupling Navier-Stokes equation and particle motion equation. In 2008, the motion of elliptical particles with neutral buoyancy in three-dimensional Poiseuille flow and the rotation and orientation characteristics of particles were studied in [20] using the fictitious domain method based on Lagrangian multipliers. Nourbakhsh *et al*. [19] studied the movement of three-dimensional deformable droplets in the plane Poiseuille flow by a finite difference/interface tracking method. Kim *et al.* [16] investigated lateral migration of three-dimensional elastic capsules under the plane Poiseuille flow by the penalty immersion boundary method.

In this paper, we use FPD method to study the interaction between fluid and solid in cylinder coordinates, which has two important features [4], viz.

(1) The particle is approximated as a highly viscous fluid.

(2) The viscosity profile is described by a smooth interface profile function.

Approximation (1) makes the method free from the solid-fluid boundary condition, significantly simplifies the treatment of many-body hydrodynamic interactions while satisfying the incompressible condition without the Stokes approximation. Approximation (2) allows to incorporate an extra degree of freedom in a fluid. Therefore, the Navier-Stokes