Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **16**, No. 2, pp. 398-422

DOI: 10.4208/aamm.OA-2021-0289
April 2024

An Analytical Approach for Buckling of FG Cylindrical Nanopanels Resting on Pasternak's Foundations in the Thermal Environment

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Received 9 September 2021; Accepted (in revised version) 29 August 2022

Abstract. In this article, the effects of temperature and size-dependent on the buckling behavior of functionally graded (FG) cylindrical nanopanels resting on elastic foundation using nonlocal strain gradient theory are investigated in detail analytical approach. According to a simple power-law distribution, the material properties of FG cylindrical nanopanels are assumed to vary continuously through the thickness direction. The Pasternak model is used to describe the reaction of the elastic foundation on the FG cylindrical nanopanels. The fundamental relations and stability equations are derived by applying the nonlocal strain gradient theory and the classical shell theory based on the adjacent equilibrium criterion. Using Galerkin's method, the mechanical buckling behavior of FG cylindrical nanopanels resting on an elastic foundation in the thermal environment is solved. The reliability of the obtained results has been verified by comparison with the previous results in the literature. Based on the obtained results, the influences of the material length scale parameter, the nonlocal parameter, temperature increment, geometric parameters, material properties, and elastic foundation on buckling behaviors of FG cylindrical nanopanels resting on an elastic foundation in the thermal environment are analyzed and discussed.

AMS subject classifications: 74K25, 74G60

Key words: Functionally graded materials, cylindrical nanopanel, nonlocal strain gradient theory, thermal, Galerkin method.

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1 Introduction

Functionally graded materials (FGMs) are a new generation of engineering materials proposed in 1984 by Japanese researchers. FGMs are usually composed of ceramic and metal so that material properties vary smoothly and continuously through the thickness from the surface to the other surface. The mechanical properties are graded in the thickness direction according to volume fraction power-law distribution, exponential distribution, or sigmoid distribution. The importance of FGMs was realized by popular applications in many fields such as tanks and pressure vessels, missiles and spacecraft, submarines, nuclear reactors, jet nozzles, and aerospace engineering structures, more and more studies focused on buckling, vibration, and dynamic responses of FGMs structures [1–4].

Nowadays, because of the brilliant properties such as mechanical properties, electrical properties, thermal properties, and other known physical and chemical properties, the nanoscale structures consist of functionally graded (FG) nanoscale structures are becoming to increase used in different fields of science and technology such as engineering, medicine, aerospace, electronics, and modern industry. Mechanical behaviors, including vibration, buckling, static deformation, and dynamic response, have a significant role in the overall operation of many nanoelectromechanical systems. For nanoscale structures, the material properties can vary from one point to another. Studying the behavior of nanoscale structures using classical theories is inaccurate because these theories ignore the size dependence and inability to describe the effects of the nanostructures size. Therefore, several size-dependent continuum theories have been proposed that could observe the size-dependent effect on the static and dynamic responses of nanoscale structures such as the nonlocal elasticity theory [5, 6], the surface elasticity [7–9], the couple stress and modified couple stress theories [10–14]. Besides, the Doublet Mechanics theory [15–17] and Energy equivalent methods [18–20] have been used to analyze stability, dynamic, and vibration of carbon nanotubes. Recently, by incorporating the effects of strain gradients and stress nonlocalities in one continuum-based theory, Lim et al. proposed the nonlocal strain gradient theory [21]. This theory can be considered to be the most generalized elasticity theory to date. This elasticity theory takes the advantages of pure nonlocal and strain gradient models, leading to a higher-order size-dependent model which can be used for a wide range of small size structure types.

Several researchers have investigated several works related to the mechanical behaviors of FG nanoscale structures using the above continuum-based models. Based on Eringen's nonlocal elasticity and Euler–Bernoulli beam theory, Ghadiriat et al. [22] presented a free vibration analysis of size-dependent FG rotating nanobeams with all surface effect considerations. The nonlinear free vibration analysis of nonlocal strain gradient nanobeams has been presented by Şimşek [23]. In this paper, the nanobeam's material properties are assumed to vary continuously in the thickness direction according to simple power-law. The basic equations and the motion equations are derivered using the nonlocal strain gradient theory and Euler-Bernoulli beam theory in conjunction with Hamilton's principle and Galerkin's approach. Mehralian and Beni [24] investigated the