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Nonconforming Finite Element Methods for Two-Dimensional Linearly Elastic Shallow Shell Model

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Abstract. A shell whose height is far less than the minimum size covering the bottom is called the shallow shell. As a branch of linear elastic shell, it is a special shell with large span and has been widely applied in engineering fields. The main aim of this paper is to construct a general nonconforming finite element framework for a two-dimensional shallow shell model proposed by Ciarlet and Miara. Based on the different regularities of the displacement components, we give the special properties satisfied by the general framework and provide several nonconforming finite element discretization schemes. Then, the existence and uniqueness of the numerical solutions are proved, with the rate of convergence derived. Finally, numerical experiments are carried out for the paraboloid, spherical dome and cylindrical bridge, which validates the theoretical analyses. Moreover, the computing cost of discretizing the shallow shell model is evidently less than that of discretizing the general shell model with comparable accuracy when the shell is the large span shell.

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1 Introduction

The linearly elastic shallow shell, a branch of the linearly elastic shell, is an important topic in elastic theory. Shallow shells are frequently used in many engineering fields, such as the roof, gymnasium, arch bridge, just name a few of them. The *locking phenomena* [1, 2], i.e., the numerical solution approaches to zero whereas the true solution does not approaches to zero when the semi-thickness ε of the shell approximates to zero, can be observed frequently in discretization of the three-dimensional (3D) model. In order to avoid this phenomenon, the two-dimensional (2D) model has attracted increasing attention recently. Using the asymptotical analysis, Ciarlet and Miara [3] proved that, as the thickness of the shell approaches zero, the solution of the 2D shallow shell equation is the H^1-limit of the solutions of scaled 3D equation. Léger and Miara [4] studied the shallow shells with single-sided contact of obstacles. Raja and Sabu [5] deduced that when the thickness of material tends to zero, the minimized energy function sequence of the 3D shallow shell model converges to the minimized energy function sequence related to the 2D model by Γ-convergence method.

For the numerical analyses, [6–8] used the conforming finite element discretization for the classical shell model and proved the error estimates. In a recent paper, we have carried out conforming element discretization for shallow shell model [9]. As we all know, one drawback of conforming element approximation is that the number of the degrees of freedom is very large and the degrees of the shape function are usually quite high. Compared with the conforming finite elements, the nonconforming element [10,11] employs fewer degrees of freedom.

The main purpose of the present work is to design efficient nonconforming finite element methods, and investigate the properties of the approximating solution. Motivated by the different smoothness of the displacement components, the conforming finite element (e.g., linear or bilinear elements) is used to approximate the first two components of the displacement, whereas the nonconforming finite element (e.g., Morley, Zienkiewicz, Fraeijs de Veubeke, Specht, rectangular Morley and ACM elements) is used to approximate the third component. It seems interesting to approximate different component of a vector field by different finite elements. Such idea has been used in problems arising from the shell model [12], Stokes flow [13], Reissner-Mindlin plate [14] and Maxwell equations [15]. Moreover, we prove that these numerical schemes have the same rate of convergence in the energy norm. It is worth mentioning that the discretization cost of shallow shell model is obviously lower than that of the Koiter's shell with comparable accuracy, which may be due to the different coordinate systems employed in these two types of models.

The rest of this paper is organized as follows. In Section 2, we prove the existence and the uniqueness of the weak solution of the 2D shallow shell model. In Section 3, we propose several finite element schemes to approximate the displacement field, and a discussion on the well-posedness of the numerical solution, and derive the error estimates for all the schemes. In Section 4, the Morley element is employed as the representative to