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# IMAGE SUPER-RESOLUTION RECONSTRUCTION BY HUBER REGULARIZATION AND TAILORED FINITE POINT METHOD*

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**Abstract**

In this paper, we propose using the tailored finite point method (TFPM) to solve the resulting parabolic or elliptic equations when minimizing the Huber regularization based image super-resolution model using the augmented Lagrangian method (ALM). The Huber regularization based image super-resolution model can ameliorate the staircase for restored images. TFPM employs the method of weighted residuals with collocation technique, which helps get more accurate approximate solutions to the equations and reserve more details in restored images. We compare the new schemes with the Marquina-Osher model, the image super-resolution convolutional neural network (SRCNN) and the classical interpolation methods: bilinear interpolation, nearest-neighbor interpolation and bicubic interpolation. Numerical experiments are presented to demonstrate that with the new schemes the quality of the super-resolution images has been improved. Besides these, the existence of the minimizer of the Huber regularization based image super-resolution model and the convergence of the proposed algorithm are also established in this paper.

*Mathematics subject classification:* 65M32, 68U10, 65K10.
*Key words:* Image super-resolution, Variational model, Augmented Lagrangian methods, Tailored finite point method.

## 1. Introduction

Image super-resolution, a process to estimate a high-resolution image from one or multiple low-resolution images, without a loss in signal-to-noise ratio, has a wide range of applications in medical imaging, high definition television, satellite imaging, synthetic aperture radar, and so on. In this paper, we mainly consider single-frame super-resolution, which is more challenging. The purpose is to obtain a high-resolution image by up-sampling a single image. In general, it is of course impossible to recover fine-scale details that are missing from low-resolution images. We can only hope to reconstruct some very specific structures, or produce visually pleasing high-frequency textures. The super-resolution restoration problem is ill conditioned since the low-resolution image we get is obtained by the original image through a convolution operator

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and a down-sampling operator. Several models and algorithms have been proposed to deal with this problem: interpolation based methods, statistics based methods, machine learning methods [1,12], least squares based methods, Fourier series based methods, and other variational methods.

Image interpolation is the most direct idea for image super-resolution problems. In the literature, the methods for implementing interpolation are different. Image interpolation can be implemented based on the digital convolution of the appropriate basis function or composition function selected, or it can be implemented using a polynomial with spatial parameters. After polynomial image interpolation, ringing effects, aliasing effects, blocking effects, and blurring effects often occur [21]. In order to achieve high-quality image interpolation results, people have carried out research on adaptive polynomial image interpolation techniques. Recently, some scholars have proposed image interpolation methods based on linear spatial changes [30]. This method estimates the biased distance between any interpolation pixel and its neighboring pixels, and shifts the interpolation pixel to a neighborhood with the same characteristics, which has a better effect in edge interpolation. When the image is based on non-ideal acquisition, a correction filter is needed before interpolation. Correction filter estimation is a morbid problem, and regularization and linear minimum mean square error (LMMSE) filters are often used, such as adaptive least squares interpolation algorithm [2], LMMSE method [14] and maximum entropy interpolation [15].

Typical machine learning methods include external example methods, artificial neural network (ANN) methods, generative adversarial network (GAN) based methods, and methods based on unrolling dynamics (UD) [10] etc. In 2010, Yang *et al.* [39] used sparse-coding-based method (SC) for image super-resolution reconstruction. Selecting a small part in a large number of data sets, as the elements to reconstruct new data, its difficulty lies in solving the objective function. The most commonly used algorithm is the gradient descent method. In 2014, Dong *et al.* [13] proposed an image super-resolution convolutional neural network (SRCNN). Unlike SC, SRCNN does not explicitly learn a dictionary or model for modeling patch space. Manifolds can improve the quality of image restoration through larger data sets or larger models. In 2016, Kim *et al.* [24] proposed a deep convolutional network model, which overcomes the disadvantages of the slow convergence rate of the SRCNN model and only targets a single scale, and uses a residual learning and high learning rate to improve the convergence rate. And they extended the work to multi-resolution image super-resolution reconstruction of a single network. In 2019, Shaham *et al.* [33] proposed the SinGAN model, an unconditionally generated adversarial model that can learn from a single natural image. It can also capture the distribution of plaque inside the image while maintaining the overall structure of the image. In 2020, Wen *et al.* [35] proposed a residual network with cascading simple blocks, abbreviated CSBRN. The main contributions include: cascading simple blocks can approximate a more complex function with fewer parameters; skip connections can help to alleviate gradient disappearance and gradient explosion; a novel loss function called detail perception loss function is used to capture more texture details; and the proposed network is divided into two pathways, which are used for low frequency information transfer and high frequency information prediction respectively. In order to make the method more effective, Fang *et al.* [16] added the image prior knowledge into the model, proposed a new edge-guided image restoration framework and developed an accurate Soft-edge assisted network (SeaNet) for image super-resolution. The SeaNet consists of three sub-nets: a rough image reconstruction network (RIRN), a soft-edge reconstruction network (Edge-Net), and an image refinement network (IRN).