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Empirical Study on Option Pricing under Markov Regime Switching Economics

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Abstract. In this research, we summarize the results of a practical study of index options based on the option valuation model which was proposed by Siu and Yang (Acta Math. Appl. Sin. Engl. Ser., 25(3) (2009), pp. 339–388), where an EMM kernel is integrated which takes into account all risk components of a regime-switching model. Further, the regime-switching risk of an economy in the options is priced using a hidden Markov regime-switching model with the risky underlying asset being modulated by a discrete-time, finite-state, hidden Markov chain whose states represent the hidden states of an economy. We apply such a model to the pricing of Hang Seng Index options based on the real-world financial data from October 2009 to October 2010 (i.e., for the year in which the model was proposed). We employed the entropy martingale measure (EMM) approach proposed by Siu and Yang (Acta Math. Appl. Sin. Engl. Ser., 25(3) (2009), pp. 339–388) to determine the optimal martingale measure for the Markov-modulated GBM. In addition, we have proposed a numerical technique called the weighted difference method to compliment the EMM approach. We have also verified the extended jump-diffusion model under regime-switching that we proposed recently (Int. J. Finan. Eng., 6(4) (2019), 1950038) using the 50ETF options which are obtained from Shanghai Stock Exchange covering a time span from January 2018 to December 2022. Further, we have highlighted the challenges for the EMM kernel-based Markov regime-switching model for pricing the out-of-the-money index options in the real world.

AMS subject classifications: 91G20, 91G60

Key words: Option pricing, EMM, regime-switching, hidden Markov model, Esscher transform, weighted difference method.

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1 Introduction

In recent years, option valuation problems under regime-switching have received considerable interest in literature. A key feature of regime-switching models is that model parameters are modulated by a Markov chain whose states represent states of business cycles (see Hamilton (1989)). Some early works on option pricing under regime-switching conditions include Naik (1993), Guo (2001), Buffington and Elliott (2002), Elliott et al. (2005), Siu (2008) and others. To be more specific, Guo (2001) investigated an option pricing problem in an incomplete market modelled by adjoining the Geometric Brownian Motion (GBM) for stock returns with a Markov chain in a Black-Scholes (1973) economy. Buffington and Elliott (2002) considered the option pricing problems for European and American options in a Black-Scholes market in which the states of the economy are described by a continuous-time, finite-state, Markov chain. Yao et al. (2003) investigated the pricing of European options under a Markov-modulated GBM and determined an equivalent martingale pricing measure for the Markov-modulated GBM. Elliott et al. (2005) proposed the use of a regime-switching version of the Esscher transform to determine an equivalent martingale measure for valuing options in a Markov-modulated Black-Scholes-Merton economy. Indeed, Gerber and Shiu (1994) pioneered the use of the Esscher transform in finance, in particular in option valuation. It provides a convenient method to specify an equivalent martingale measure. Siu (2008) justified the use of the Esscher transform for option valuation in a regime-switching diffusion model and a regime-switching jump-diffusion model using a game theoretic approach. Siu and Yang (2009) considered a modified version of the Esscher transform used in Elliott et al. (2005) to incorporate explicitly the intensity matrix of the Markov chain in the specification of an equivalent martingale measure. Siu (2011) demonstrated, through a rigorous mathematical proof, that an optimal equivalent martingale measure selected by minimizing the relative entropy between an equivalent martingale measure and the real-world probability measure does not price the regime-switching risk. Elliott et al. (2013) investigated the pricing of both European and American-style options when the price dynamics of the underlying risky assets are governed by a Markov-modulated constant elasticity of variance process. Liu (2017) conducted an empirical study using Markov-modulated regime switching model on Hang Seng index options when the regime switching risk is priced. In recent years, regime-switching models have been extended to include a jump-diffusion process (Momeya, e al., 2016), or price different types of options, for instance, bond options (Shen, et al., 2013), currency options (Bo, et al., 2010; Liu, 2019), and foreign equity options (Lian, et al., 2016; Fan, et al., 2014).

In terms of option valuation principles, it has been established (see, Harrison