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# The Existence and Global $p$-Exponential Stability of Periodic Solution for Stochastic BAM Neural Networks with Delays*

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**Abstract** In this paper, we consider a class of stochastic BAM neural networks with delays. By establishing new integral inequalities and using the properties of spectral radius of nonnegative matrix, some sufficient conditions for the existence and global $p$-exponential stability of periodic solution for stochastic BAM neural networks with delays are given. An example is provided to show the effectiveness of the theoretical results.

**Keywords** Global $p$-exponential stability, periodic solution, BAM neural networks, stochastic

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## 1. Introduction

A class of two-layer interassociative networks called bidirectional associative memory (BAM) neural networks is an important model with the ability of information memory and information association, which is crucial for application in pattern recognition, solving optimization problems and automatic control engineering [11, 14, 18]. In such applications, the dynamical characteristics of networks play an important role.

As is well known, in both biological and man-made neural networks, delays occur due to finite switching speed of the amplifiers and communication time. They slow down the transmission rate and can influence the stability of designed neural networks by creating oscillatory or unstable phenomena. Many authors have obtained interesting results on the stability of neural networks in [4, 20, 25], and synchronization in [5], so it is more important in accordance with this fact to study the BAM neural networks with delays. The circuits diagram and connection pattern implementing for the delayed BAM neural networks can be found in [3]. In recent years, some useful results on the dynamical behaviors of the delayed BAM neural networks have been given, for example, see [12, 16, 21, 27, 28] for stability, see [23] for the synchronization, and see [2, 3, 17, 19] for the periodic oscillatory behavior. Of those, since it has been found applications in learning theory [22], which is moti-

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vated by the fact that learning usually requires repetition, it is of prime importance to study periodic oscillatory solutions of neural networks.

However, a real system is usually affected by external perturbations which in many cases are of great uncertainty and hence may be treated as random, as pointed out by [8] that in real nervous systems synaptic transmission is a noisy process brought on by random fluctuations from the release of neurotransmitters, and other probabilistic causes. Therefore, it is significant and of prime importance to consider stochastic effects to the dynamics behavior of stochastic BAM neural networks with delays. Many interesting results on stability of stochastic BAM neural networks with delays have been reported, see [1,6,15,24,29,30].

To the best of our knowledge, some authors have considered the stability of trivial solution to the stochastic BAM neural networks, see [6,15,24,29]. However, few authors have considered the existence of periodic solution to stochastic BAM neural networks with delays. Motivated by the above discussions, we will study the existence and global $p$-exponential stability of periodic solution for stochastic BAM neural networks with delays. By establishing new integral inequalities and using the properties of spectral radius of nonnegative matrix, some sufficient conditions for the existence and global $p$-exponential stability of periodic solution for stochastic BAM neural networks with delays are given. An example is provided to show the effectiveness of the theoretical results.

## 2. Model description and preliminaries

For the sake of convenience, we introduce several notations and recall some basic definitions.

Let $R^l$ ($R^l_+$) be the space of $l$-dimensional (nonnegative) real column vectors, and $R^{m\times l}$ ($R^{m\times l}_+$) denotes the set of $m \times l$ (nonnegative) real matrices. Usually $I$ denotes an $l \times l$ unit matrix. For $A,\ B \in R^{m\times l}$ or $A,\ B \in R^l$, the notation $A \geq B$ ($A > B$) means that each pair of corresponding elements of $A$ and $B$ satisfies the inequality " $\geq (>)$". Especially, $A \in R^{m\times l}$ is called a nonnegative matrix if $A \geq 0$, and $z \in R^l$ is called a positive vector if $z > 0$. Let $\rho(A)$ denote the spectral radius of nonnegative square matrix $A$.

$C(X,Y)$ denotes the space of continuous mappings from the topological space $X$ to the topological space $Y$. Especially, let $C \triangleq C([-\tau,0], R^l)$ with a norm $\|\varphi\| = \sup\limits_{-\tau\leq s\leq 0} |\varphi(s)|$ and let $|\cdot|$ be the Euclidean norm of a vector $x \in R^l$, where $\tau$ is a positive constant. Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfy the usual conditions ($i.e$, it is right continuous and $\mathcal{F}_0$ contains all P-null sets). If $x(t)$ is an $R^l$-valued stochastic process on $t \in [-\tau, \infty)$, we let $x_t = x(t+s) : -\tau \leq s \leq 0$, which is regarded as a $C$-valued stochastic process for $t \geq 0$. Denote by $BC^b_{\mathcal{F}_0}\left([-\tau,0], R^l\right)$ the family of all bounded $\mathcal{F}_0$-measurable, $C$-valued random variables $\phi$, satisfying $\|\phi\|^p_{L^p} = \sup\limits_{-\tau\leq s\leq 0} E|\phi(s)|^p < \infty$ , where $Ef$ means the mathematical expectation of $f$.

For any $x \in R^l$, $\phi \in C$, we define $[x]^+ = (|x_1|, \dots, |x_l|)^T$,and $[\phi(t)]^+_\tau = (|\phi_1|_\tau, \dots, |\phi_l|_\tau)^T$, where $|\phi_i|_\tau = \sup_{-\tau\leq s\leq 0} |\phi_i(t+s)|$, $i = 1, 2, \dots, l$.