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Existence and Approximate Controllability of Solutions for an Impulsive Evolution Equation with Nonlocal Conditions in Banach Space*

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Abstract In this article, we study the existence of mild solutions and approximate controllability for non-autonomous impulsive evolution equations with nonlocal conditions in Banach space. The existence of mild solutions and some conditions for approximate controllability of these non-autonomous impulsive evolution equations are given by using the Krasnoselskii's fixed point theorem, the theory of evolution family and the resolvent operator. In particular, the impulsive functions are supposed to be continuous and the nonlocal item is divided into Lipschitz continuous and completely bounded. An example is given as an application of the results.

Keywords Impulsive evolution equation, approximate controllability, nonlocal conditions, resolvent operator, evolution family

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1. Introduction

Recently, the evolution equation is used to describe the state or process that changes with time in physics, mechanics or other natural sciences. It is well known that the nonlocal problems are more widely used in applications than the classical ones. Byszewski [1] first investigated the nonlocal problems. They obtained the existence and uniqueness of mild solutions for nonlocal differential equations without impulsive conditions. Deng [3] pointed out that the nonlocal initial condition can be applied in physics with a better effect than the classical initial condition $u(0) = u_0$ and used the nonlocal conditions $u(0) = \sum_{k=1}^{m} c_k u(t_k)$ to describe the diffusion

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phenomenon on a small amount of gas in a transparent tube. The above findings have encouraged more authors to focus on differential equations with non-local conditions. The differential equations with non-local conditions are often applied to models of processes subject to abrupt changes in a specific time. They have a wide range of applications in areas such as control, mechanical, electrical engineering fields and so on. Fan [4] discussed the existence results for semilinear differential equations with nonlocal and impulsive conditions. Tai [5] studied the exact controllability of a fractional impulsive neutral functional integro-differential systems with nonlocal conditions by using the fractional power of operators and the Banach contraction mapping theorem. When describing some physical phenomena, the nonlocal condition can be more useful than the standard initial condition $u(0) = u_0$. The importance of nonlocal conditions has also been discussed in [1,2,5,6,14–17,19,22–28,30,31]. When discussing the problem of evolution equations, it is often necessary to explore their application and combine them with the controllability.

The controllability and approximate controllability of evolution equations are considered by many authors owing to their wide applications in the field of physics, biology and medicine, see [5,6,9–14,17,21,29]. The concept of controllability, after being first introduced by Kalman [7] in 1963, has become an active area of research due to its enormous applications in physics. There are various studies on the approximate controllability of systems represented by differential equations, integral differential equations, differential inclusion, neutral-type generalized differential equations and integer-order impulsive differential equations in Banach spaces. Mahmudov [8] in 2008 studied the approximate controllability for the abstract evolution equations with nonlocal conditions in Hilbert spaces and obtained sufficient conditions for the approximate controllability of the semi-linear evolution equation. In 2018, Chen [12] discussed the approximate controllability of non-autonomous evolution system with nonlocal conditions and introduced a new Green's function to prove the existence of mild solutions. The approximate controllability of the development equation with impulse makes the application of the development equation more practical and representative.

Impulsive differential equations are commonly used for modelling processes that change abruptly at some point in time. They have a wide range of applications in control, mechanical, electrical and other fields. These changes of state are caused by transient forces (perturbations). Differential systems that use transient forces as impulsive conditions appear in many applications, such as biological phenomena involving thresholds, sudden rhythm models in medicine and biology, optimal control models in economics and frequency modulation systems. For these reasons, Hernández and O'Regan [9] discussed on a new class of abstract impulsive differential equations, introduced a new model named as differential equations without instantaneous impulses. It shows that the action of drugs in the blood and their absorption into the body is a gradual and continuous process. Wang [10] studied a general impulsive evolution equation and discussed periodic solutions and Ulam's type stability to a new generalized evolution equation without instantaneous impulses in the infinite-dimensional spaces.

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