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An Efficient High-Order Gas-Kinetic Scheme with Hybrid WENO-AO Method for the Euler and Navier-Stokes Solutions

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Abstract. The high-order gas-kinetic scheme (HGKS) features good robustness, high efficiency and satisfactory accuracy,the performance of which can be further improved combined with WENO-AO (WENO with adaptive order) scheme for reconstruction. To reduce computational costs in the reconstruction procedure, this paper proposes to combine HGKS with a hybrid WENO-AO scheme. The hybrid WENO-AO scheme reconstructs target variables using upwind linear approximation directly if all extreme points of the reconstruction polynomials for these variables are outside the large stencil. Otherwise, the WENO-AO scheme is used. Unlike combining the hybrid WENO scheme with traditional Riemann solvers, the troubled cell indicator of the hybrid WENO-AO method is fully utilized in the spatial reconstruction process of HGKS. During normal and tangential reconstruction, the gas-kinetic scheme flux not only needs to reconstruct the conservative variables on the left and right interfaces but also to reconstruct the derivative terms of the conservative variables. By reducing the number of times that the WENO-AO scheme is used, the calculation cost is reduced. The high-order gas-kinetic scheme with the hybrid WENO-AO method retains original robustness and accuracy of the WENO5-AO GKS, while exhibits higher computational efficiency.

AMS subject classifications: 52B10, 65D18, 68U05, 68U07

Key words: Gas-kinetic scheme, hybrid WENO-AO scheme, high-order finite volume scheme, extreme point.

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1 Introduction

The development of aerospace technology has accentuated the need to describe complex and detailed flow fields around high-speed aircrafts efficiently. The research focus in computational fluid dynamics (CFD) [1] has shifted towards high-resolution and high-order schemes. In recent years, emphasis has been placed on high-order finite volume methods [2], discontinuous Galerkin (DG) methods, and correction procedures via reconstruction (CPR) for numerical simulations involving complex geometric meshes. The gas-kinetic scheme is a numerical method that provides solutions to the Euler and Navier-Stokes equations within the finite volume framework. Its interface flux is based on a time evolution solution of the kinetic model equation, such as the Bhatnagar-Gross-Krook (BGK) model. Theoretically, it identifies more detailed small scale particle dynamics, such as on the scales of particle mean free path and mean particle collision time. The GKS is based on the kinetic modeling for the numerical flux evaluation and updates the macroscopic flow variables directly. The high-order gas-kinetic scheme (HGKS) has demonstrated excellent performance in compressible flows [3,4]. It has also been successfully extended into DG [5] methods and CPR [6] frameworks. In addition, the high-order gas-kinetic scheme and high-order gas-kinetic compact schemes [19–21] are preferred on unstructured meshes due to their higher resolution and stability.

The gas-kinetic scheme employs a time-evolving gas distribution function to reveal the multi-scale physical flow mechanism, encompassing kinetic particle transport to hydrodynamic wave propagation [7–9]. A two-stage fourth-order HGKS scheme was proposed by Pan et al. [11] based on the two-stage fourth-order generalized Riemann problem (GRP) scheme [10], becoming increasingly popular. The combination of multi-stage multi-derivative method (MSMD) and HGKS using second-order and third-order gas distribution functions resulted in two-stage fifth-order and three-stage fifth-order HGKS [12], with higher efficiency than the traditional Riemann solver [13] with Runge-Kutta (RK) time stepping method. Fifth-order WENO scheme was proposed by Jiang and Shu [14], providing a general framework for designing smoothness indicators and nonlinear weights. A fundamental concept of WENO schemes is to use a linear combination of lower order numerical fluxes or reconstructions to obtain a higher order approximation. WENO schemes employ a local characteristic decomposition method to avoid spurious oscillations. For spatial reconstruction, the classic HGKS with WENO reconstruction [7] is similar to the traditional high-order finite volume method (FVM) [15]. But the classic HGKS with WENO reconstruction [7] better captures shear instability due to the multidimensional properties of the gas distribution function [16]. Using fifth-order WENO-JS or WENO-Z reconstruction [17], non-equilibrium state and equilibrium state in the gas distribution function were generally reconstructed separately. The WENO adaptive order (WENO-AO) method proposed by Balsara [18] recently was for HGKS spatial reconstruction, overcoming shortcomings of the classic HGKS [7]. Compared to the classic WENO5-GKS scheme, HGKS with WENO-AO reconstruction has shown significant improvements in performance and no false overshoots or undershoots in certain