INTERNATIONAL JOURNAL OF
NUMERICAL ANALYSIS AND MODELING, *SERIES B*
Volume 3, Number 4, Pages 371–387

STATISTICAL PROPERTIES OF SEMIPARAMETRIC ESTIMATORS FOR COPULA-BASED MARKOV CHAIN VECTORS MODELS

WENDE YI AND B. STEPHEN SHAOYI LIAO

Abstract. This paper proposes a method for estimation of a class of copula-based semiparametric stationary Markov vector time series models, namely, the two-stage semiparametric pseudo maximum likelihood estimation (2SSPPMLE). These Markov vector time series models are characterized by nonparametric marginal distributions and parametric copula functions of temporal and contemporaneous dependence, while the copulas capture two classes of dependence relationships of Markov time series. We provide simple estimators of marginal distribution and two classes of copulas parameters and establish their asymptotic properties following conclusions in Chen and Fan (2006) and some easily verifiable conditions. Moreover, we obtain the estimation of conditional moment and conditional quantile functions for the bivariate Markov time series model.

Key words. Copula, Semiparametric estimation, Temporal dependence, Contemporaneous dependence, and 2SSPPMLE

1. Introduction

In many fields, including international asset pricing, portfolio diversification and risk management, et al., dependence of models on random variables is an interesting topic. In dependence analysis of time series, we must synchronously consider two crucial classes of dependence relationships, namely, temporal dependence and contemporaneous dependence. However, a great deal of research has shown that economic and financial multivariate time series are typically nonlinear, are non-normally distributed and have nonlinear comovements beyond the first two conditional moments. In recent years, copulas have started to be applied to model the dependence structure of time series in various fields. In particular, modeling and estimating the dependence structure between several univariate time series in the finance and insurance community are of great interest; see Joe [1]and Embrechts et al. [2] for reviews. Chen and Fan [3], Abegaz and Naik-Nimbalkar [4] studied the properties of estimators of semiparametric and parametrical stationary Markov chain models. Embrechts et al. [2] used copula functions to model multivariate distribution of returns. In economic and financial applications, it is interesting to estimate or forecast certain features of time series, such as the value-at-risk (VaR) of a portfolio of assets, which has become routine in risk management. Duffie and Pan [5], Hull and White [6], Engle and Manganelli [7] and Embrechts [8] have done much work on this topic. In Yi and Liao [9], we established a model based on copula functions to study the two classes of dependence relationships (including nonlinear dependence) of time series vectors, proposed a three-stage pseudo maximum likelihood estimation of the model and discussed the properties of parametric

Received by the editors August 2, 2012 and, in revised form October 24, 2012.
2000 *Mathematics Subject Classification.* 62F12.
This work has been supported by the National Natural Science Foundation of China (No: 71271227), the Humaniores & Social Science Research Project of the Ministry of Education (No: 11XJC790004) and the GRF grant (No: CityU 147407) of the Research Grant Commission, Hong Kong SAR Government.

estimators. In this paper, we study a class of bivariate copula-based semiparametric stationary Markov models in which temporal copulas and contemporaneous copulas are parameterized, but the marginal distributions are left unspecified. Models in this class are completely characterized by three unknown parameters, namely (1) contemporaneous dependence copula parameters θ^* (i.e., the finite-dimensional parameters to capture the dependence structure between the two univariate time series); (2) temporal dependence copula parameters $\delta^* = (\delta_X^*, \delta_Y^*)$ (i.e., the finite-dimensional parameters to characterize the temporal dependence of individual time series); and (3) marginal distributions $F^*(\cdot)$ and $G^*(\cdot)$. In estimation of the two classes of parameters of temporal and contemporaneous dependence copulas, our main contribution is that we propose a two-stage semiparameters pseudo maximum likelihood estimation (2SSPPMLE) for bivariate copula-based semiparametric time series vector models based on the two-stage estimator proposed for bivariate and univariate copula models with i.i.d. observations (Genest [10], Shih [11], Chen and Fan [3], and Abegaz and Naik- Nimbalkar [4]). Moreover, we focus on establishing the consistency and asymptotic normality of the resulting estimators of model parameters based on the 2SSPPMLE. The remainder of this paper is organized as follows. Section 2 reviews related research and proposes the models of bivariate stationary Markov time series vectors of order 1. Section 3 proposes the 2SSPPMLE of model parameters and estimations of conditional moment and conditional quantile functions copula-based bivariate Markov chain model. Section 4 presents some assumptions of consistency and asymptotical normality of semiparametric estimators of univariate copula-based time series models. Section 5 discusses the consistency and asymptotic normality of semiparametric estimators of the 2SSPPMLE. Section 6 discusses the problem of copula selection under the three-stage procedure, and the last section consists of concluding remarks.

2. Related Work and Copula-based bivariate Markov chains models of order 1

A copula is defined as a multivariate distribution with standard uniform marginal distributions. The Sklar's theorem (Sklar [12]) illustrates how to model a multivariate distribution by modeling its marginal distributions and its copula function separately (see Nelsen [13] for details).

Sklar's theorem. Let $H(\cdot,\cdot)$ be a bivariate function with continuous marginals $F(\cdot)$ and $G(\cdot)$. There exists a unique copula $C(\cdot,\cdot)$ such that the joint distribution can be written as

$$H(x,y) = C(F(x), G(y)) \qquad (x,y) \in \mathrm{R}^2$$

Research on dependence of time series based on copulas mainly focuses on two forms of dependence structures. The first is the contemporaneous dependence of multivariate time series, where the focus is on modeling the joint distribution of some random vector, $X_t = [X_{1t}, X_{2t}, \cdots, X_{nt}]'$, conditional on the information set F_{t-1}. Bouy et al. [14] used parametric copulas to model dynamic dependence of time series and provided applications to financial returns and transactions based forex data. They applied the two-step procedure of Genest et al. [15] to estimate the copula dependence parameter. Patton [16, 17] defined a "conditional copula" as a multivariate distribution of variables distributed as uniform (0,1) conditional on F_{t-1}, and estimated parameters of multivariate models for time series of possibly different lengths.

The second is the temporal dependence of univariate time series based on stationary Markov chains, where copulas are used to describe the dependence between