INTERNATIONAL JOURNAL OF
NUMERICAL ANALYSIS AND MODELING
Volume 14, Number 2, Pages 255–282

HIGH-RESOLUTION IMAGE RECONSTRUCTION: AN env_{ℓ^1}/TV MODEL AND A FIXED-POINT PROXIMITY ALGORITHM

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Abstract. High-resolution image reconstruction obtains one high-resolution image from multiple low-resolution, shifted, degraded samples of a true scene. This is a typical ill-posed problem and optimization models such as the ℓ^2/TV model are previously studied for solving this problem. It is based on the assumption that during acquisition digital images are polluted by Gaussian noise. In this work, we propose a new optimization model arising from the statistical assumption for mixed Gaussian and impulse noises, which leads us to choose the Moreau envelop of the ℓ^1-norm as the fidelity term. The developed env_{ℓ^1}/TV model is effective to suppress mixed noises, combining the advantages of the ℓ^1/TV and the ℓ^2/TV models. Furthermore, a fixed-point proximity algorithm is developed for solving the proposed optimization model and convergence analysis is provided. An adaptive parameter choice strategy for the developed algorithm is also proposed for fast convergence. The experimental results confirm the superiority of the proposed model compared to the previous ℓ^2/TV model besides the robustness and effectiveness of the derived algorithm.

Key words. High-resolution image reconstruction, env_{ℓ^1}/TV model, proximity operator, fixed-point algorithm.

1. Introduction

High-resolution (HR) image reconstruction arises from many applications, such as remote sensing, surveillance, and medical imaging. HR image provides high pixel density and rich image details which are desired in many image-related areas. For example, HR tomography images can help doctors to diagnose nidus at an early stage, and HR remote sensing images contain rich spectrum and spatial information, such as texture and shapes. However, due to the limitation of solid-state sensors such as charge-coupled devices (CCD) and complementary metal oxide semiconductors (CMOS), the way of improving hardware resolution is expensive, and sometimes hardly meets the need of better image details and greater image clarity. HR image reconstruction is an approach that reconstructs one HR image from multiple shifted, degraded low-resolution (LR) images. It can break through the resolution limit of sensor manufacturing techniques and reduce the cost of high precision optics and imaging sensors based on existing imaging equipments.

The image degradation procedure involves blurring, downsampling, displacement-error and noise. The observation model can be written as [25]

$$c = Au + \eta, \tag{1}$$

where c is an observed LR image, η is the noise, u denotes the desired HR image, and A is the degradation system matrix. The observation model (1) can be specified with the estimated registration parameters and the given boundary condition. In this paper we focus on a specific HR image reconstruction problem based on a mathematical model for a prefabricated multi-sensors image acquisition system proposed in [9], which will be explained in section 2. This model is feasible to acquire subpixel image information by placing sensors in coupled shifted subpixel positions.

Received by the editors on January 6 ,2016 and accepted on October 14, 2016.
2000 *Mathematics Subject Classification.* 35R35, 49J40, 60G40.

The HR image reconstruction is an ill-posed problem due to the compactness of the degradation operators [45]. Regularization methods are an effective way to obtain a stable approximate solution. Regularization methods for HR reconstruction can be roughly divided into stochastic and deterministic approaches. Stochastic HR approach is based on statistical modeling of noise and image degradation process [51, 52, 53, 54]. Previous studies showed that the stochastic approach has major advantage in robustness. The deterministic approach exploits the prior assumption of images and reconstructs HR images via an optimization framework [1, 3, 42]. However, if there is no or lack of prior information, it is difficult to develop a suitable optimization model.

In considering the image degradation factors, the noise is sometimes modeled as Gaussian white noise. Previously, the ℓ^2/TV model was applied to reconstruct HR images with the ℓ^2-norm as fidelity term describing the model error [43]. Meanwhile, in real applications, the imaging acquisition systems may suffer from impulse noise which is usually caused by malfunctioning arrays in camera sensors, faulty memory locations in hardware, or transmission in a noise channel [11, 60]. In this case, the ℓ^2/TV model may perform poorly because the ℓ^2-norm has less ability to reduce the effect of outliers compared to the ℓ^1-norm [10, 12, 42, 47, 57]. Moreover, the estimated displacement-errors should be specifically considered during the reconstruction via a regularization method.

The first contribution of this paper is to propose a robust optimization model arising from a statistical assumption for mixed Gaussian and impulse noise, denoted as the env_{ℓ^1}/TV model, by replacing the fidelity term in the ℓ^2/TV model with the Moreau envelop of the ℓ^1-norm. It is based on the observation that the Moreau envelop of the ℓ^1-norm balances the advantages of the ℓ^1-norm and the ℓ^2-norm. Some researchers tried to combine the ℓ^1-norm and the ℓ^2-norm for the fidelity term [50, 58, 59]. However, the choice strategy of the tradeoff parameter between these two norms is not clear and implementing a choice strategy is time consuming [59]. In this study, we consider the joint distribution of Gaussian white noise and impulse noise. Using segmental maximum a posteriori (MAP) criteria, we derive the Moreau envelop of the ℓ^1-norm to define the fidelity term. The Moreau envelop of the ℓ^1-norm has some desired properties. First, the Moreau envelop of the ℓ^1-norm is differentiable and its derivative has a closed form. Second, the parameter appearing in the Moreau envelop has statistical meaning, and is related to the Gaussian noise level and incident probability of impulse noise, which can be estimated from images iteratively. In our experiment we propose a choice strategy of τ adaptively according to the number of iterations. The numerical result shows that the performance of the proposed model is stable for mixed Gaussian and impulse noise.

The second contribution of this paper is to propose a fixed-point proximity algorithm to solve the model that avoids finding the inverse of $A^T A$. In this paper, we focus on solving the models regularized by the TV-norm. It is difficult to minimize by conventional methods due to the non-smoothness of the TV regularization term. A number of numerical methods have been proposed to address this issue, such as the primal-dual method [13, 20, 24, 35, 61], the alternating direction method of multipliers [23], interior point algorithms [30] and fixed-point methods [19, 33, 36]. These algorithms have been broadly used in solving image deblurring problem and were reported with great performance. Most of the algorithms mentioned above require computing the inverse of $A^T A$. When facing the HR image reconstruction with the displacement-error problem, the fact that the size of matrix A is large and there is no fast algorithm (like FFT and DCT) to efficiently compute the inverse of $A^T A$ would increase the computational cost. In this paper, we propose a new fixed-point algorithm that