Anal. Theory Appl.
Vol. 27, No. 1 (2011), 92–100
DOI10.1007/s10496-011-0092-9

THE BOUNDEDNESS OF BILINEAR SINGULAR INTEGRAL OPERATORS ON SIERPINSKI GASKETS

Ming Xu and Shengmei Wang
(*Jinan University, China)*

Received May 25, 2010; Revised Nov. 15, 2010

Abstract. In the paper we give the boundedness estimate of bilinear singular integral operators on Sierpinski gasket inspired from [1].

Key words: *boundedness, bilinear singular integral, Sierpinski gaskets*

AMS (2010) subject classification: 42B20

1 Introduction

In [1], V. Chousionis studied the boundedness of a class of singular integral operators associated with homogeneous Calderón-Zygmund standard kernels on Sierpinski gaskets. In [1], the author mainly cares about such singular integral operator

$$T_{\lambda,\varepsilon}f(x)=\int_{|x-y|>\varepsilon}\frac{\Omega_\lambda((x-y)/|x-y|)}{h_\lambda(x-y)}f(y)\mathrm{d}\mu_\lambda(y),$$

where μ_λ is the restriction of the d-dimensional Hausdorff measure on λ-Sierpinski gasket E_λ for $d_\lambda=-\dfrac{\log 3}{\log\lambda}$ and $\lambda\in(0,1/3)$, here $\Omega_\lambda(\cdot)$ is an odd function defined on the unit sphere S^1 and $h_\lambda(\cdot)$ satisfies some kind of increasing conditions. In fact, a kind of T(1) theorem[4] on Sierpinski gasket is given in [1]. As we know, in a series of papers, L. Grafakos and R.H. Torres(see[5][6][7][8] etc) gave a version of T(1) theorem for multilinear singular integrals in Euclidean space. Here naturally we have one problem: What is about the boundedness of multilinear singular integrals correponding to [1] on Sierpinski gaskets?The purpose of the paper is to give a complete answer of the problem.

In fact, Sierpinski gaskets considered in the paper with suitable metric can be seen as a space of homogenous type. The multilinear T(1) theorem on a space of homogenous type can be given directly as an extension of the case in Euclidean geometric structure. But in the process it seems that we lose some speciality of fractal structure. Thus it's necessary to study such kind of problems so that we can know deeply the speciality of some fractal structure which leads to some speciality of the boundedness of singular integral operators.

The paper is organized as follows: In section 2, we give the notion of some Sierpinski gaskets and the definition of multilinear singular integral operator, for simplicity, we only consider the bilinear operator. At last we give the statement of main theorem in the paper; In section 3, we give the proof of main theorem.

Through out the paper, the constant "C" and "c" may be different somewhere, but it is not essential.

2 Some Important Lemmas and Main Theorem

The following notations come from the corresponding part in [1]. For $\lambda \in (0,1/3)$, SG_λ can be achieved by the following similitude $s_i^\lambda : \mathbf{R}^2 \to \mathbf{R}^2 (i=1,2,3)$,

(1) $s_1^\lambda(x,y) = \lambda(x,y)$;

(2) $s_2^\lambda(x,y) = \lambda(x,y) + (1-\lambda, 0)$;

(3) $s_3^\lambda(x,y) = \lambda(x,y) + (\frac{1-\lambda}{2}, \frac{\sqrt{3}}{2}(1-\lambda)$.

For $\alpha \in I^n$, say $\alpha = (i_1, \dots, i_n)$, define $s_\alpha^\lambda : \mathbf{R}^2 \to \mathbf{R}^2$ through the iteration

$$s_\alpha^\lambda = s_{i_1}^\lambda \circ s_{i_2}^\lambda \cdots \circ s_{i_n}^\lambda.$$

Let A be the equilateral triangle with vertices $(0,0),(1,0),(1/2,\sqrt{3}/2)$. Denote $s_\alpha^\lambda(A) = S_\alpha^\lambda$, $I^0 = \{0\}$ and $s_0^\lambda = id$. The limit set of the iteration can be given by

$$E_\lambda = \bigcap_{j\geq 0} \bigcup_{\alpha \in I^j} S_\alpha^\lambda$$

with Hausdorff dimension $d_\lambda = -\frac{\log 3}{\log \lambda}$. The measure μ_λ is the restriction of Hausdorff measure to E_λ, which is d_λ-AD regular, that is,

$$\mu(B(x,r)) \sim r^{d_\lambda}, \tag{2.1}$$

where $B(x,r)$ is a ball with center $x \in E_\lambda$ and $0 < r \leq 1$. Also here let $\alpha \in I^n, \beta \in I^k$, set $\beta\lfloor n = \alpha$ to denote the restriction of β in its first n coordinates α.

From the condition (2.1), it's easy to know that $(E_\lambda, \rho, \mu_\lambda)$ is a space of homogeneous type (see [9]), where $\rho(x,y) = \inf\{r > 0 : y \subset B(x,r)\}$ is a quasi-metric function associated to μ_λ. In