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CONVERGENT FINITE DIFFERENCE SCHEME FOR 1D FLOW OF COMPRESSIBLE MICROPOLAR FLUID

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Abstract. In this paper we define a finite difference method for the nonstationary 1D flow of the compressible viscous and heat-conducting micropolar fluid, assuming that it is in the thermodynamical sense perfect and polytropic. The homogeneous boundary conditions for velocity, microrotation and heat flux are proposed. The sequence of approximate solutions for our problem is constructed by using the defined finite difference approximate equations system. We investigate the properties of these approximate solutions and establish their convergence to the strong solution of our problem globally in time, which is the main results of the paper. A numerical experiment is performed by solving the defined approximate ordinary differential equations system using strong-stability preserving (SSP) Runge-Kutta scheme for time discretization.

Key words. micropolar fluid flow, initial-boundary value problem, finite difference approximations, strong and weak convergence.

1. Introduction

The theory of micropolar fluid was introduced by A. C. Eringen in 1960, [8]. Eringen suggested many possible applications of the micropolar fluid, but from the mathematical point of view the theory is still in the early stage of development. The results for incompressible flow are very well systematized in the book of Lukaszewicz,[11] but the theory for compressible flows, especially for the flows involving temperature, is still in the beginning.

In this paper we focus on the compressible flow of the isotropic, viscous, and heat conducting micropolar fluid, which is in thermodynamical sense perfect and polytropic. The model for this type of flow was first considered by Mujaković in [12] where she developed a one-dimensional model. The model is quite complex from numerical point of view, as well as from theoretical standpoint. It consists of four partial differential equations - one of which is a differential equation of the first order, and the other three are non-linear parabolic equations of second order. In the work [13] the local existence and uniqueness of the solution, which is called generalized, for our model with the homogeneous boundary conditions for velocity, microrotation and heat flux were proved, while in [13] Mujaković proved the existence of global in time solution for the described problem. So far, the numerical analysis of this model was done only by Faedo-Galerkin method [12, 7, 15] that is unsuitable for wider application.

The main goal of this paper is to propose a numerical method for solving a given model using the finite difference approach, which is more acceptable in practical applications. We define the semidiscrete finite difference approximate equations system and investigate the properties of the sequence of the approximate solutions. We prove that the limit of this sequence is the solution to our problem and that it has the same properties as the solution in [12]. In this way the convergence

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of the corresponding numerical scheme is established and furthermore, the global existence of the solution for the considered problem, already proved in [13], verified. In our work we follow some ideas of [3, 4].

Other authors who have discussed various models of fluid by using finite differences mainly don't analyze the problem of convergence of approximate solutions from a theoretical point of view. The approach used here can be applied in other research models based on similar systems of partial differential equations.

The paper is organized as follows. In the second section we introduce the mathematical formulation of our problem. In the third section we derive the finite difference approximate equations system and in the fourth section present the main result. In Sections 5-8, we prove uniform a priori estimates for the approximate solutions. Proof of convergence of a sequence of approximate solutions to a solution of our problem is given in the ninth section. Finally, in the tenth section we perform the numerical experiment.

2. Mathematical model

We are dealing with the one-dimensional flow of the compressible viscous and heat-conducting micropolar fluid flow, which is thermodynamically perfect and polytropic. Let ρ, v, w and θ denote, respectively, the mass density, velocity, microrotation velocity and temperature in the Lagrangian description. The motion of the fluid under consideration is described by the following system of four equations (see, for example, [12]):

$$\partial_t\rho + \rho^2\partial_x v = 0, \tag{1}$$

$$\partial_t v = \partial_x\left(\rho\,\partial_x v\right) - K\partial_x(\rho\,\theta), \tag{2}$$

$$\rho\,\partial_t\omega = A\left[\rho\,\partial_x\left(\rho\,\partial_x\omega\right) - \omega\right], \tag{3}$$

$$\rho\,\partial_t\theta = -K\rho^2\theta\,\partial_x v + \rho^2(\partial_x v)^2 + \rho^2(\partial_x\omega)^2 + \omega^2 + D\rho\,\partial_x\left(\rho\,\partial_x\theta\right). \tag{4}$$

The system is considered in the domain $Q_T = (0,1)\times(0,T)$, where $T>0$ is arbitrary; K, A and D are positive constants. Equations (1)-(4) are, respectively, local forms of the conservation laws for the mass, momentum, momentum moment and energy. We take the following non-homogeneous initial conditions:

$$\rho(x,0)=\rho_0(x),\ v(x,0)=v_0(x),\ \omega(x,0)=\omega_0(x),\ \theta(x,0)=\theta_0(x), \tag{5}$$

and homogeneous boundary conditions:

$$v(0,t)=v(1,t)=0, \qquad \omega(0,t)=\omega(1,t)=0 \tag{6}$$

$$\partial_x\theta(0,t) \;=\; \partial_x\theta(1,t)=0, \tag{7}$$

for $x\in(0,1)$ and $t\in(0,T)$. Here ρ_0, v_0, ω_0 and θ_0 are given functions. We assume that there exists a constant $m\in\mathrm{R}^+$ such that

$$\rho_0(x)\ge m, \quad \theta_0(x)\ge m \quad \text{for } x\in(0,1). \tag{8}$$

Let the initial data (5) have the following properties of smoothness

$$\rho_0,\theta_0\in H^1((0,1)) \quad\text{and}\quad v_0,\omega_0\in H_0^1((0,1)). \tag{9}$$

Because of embedding $H^1((0,1))$ into $C([0,1])$, it is easy to check that there exists $M\in\mathrm{R}^+$ such that

$$\rho_0(x), |v_0(x)|, |\omega_0(x)|, \theta_0(x)\le M, \quad \text{for } x\in[0,1]. \tag{10}$$

Under the stated assumptions (8)-(9) in the previous papers [12, 13] is proven that problem (1)-(7) has unique solution (ρ,v,ω,θ) in the domain Q_T, for every $T>0$,