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NUMERICAL INVERSION SCHEMES FOR MAGNETIZATION USING AEROMAGNETIC DATA

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Abstract. The re-weighted regularized conjugate gradient (RRCG) method has been a popular algorithm for magnetic inversion problems. In this work, we show that for a two-dimensional problem with uniform field data, the resulting coefficient matrix to be inverted has a symmetric Block-Toeplitz Toeplitz-Block (BTTB) structure. Taking advantage of the BTTB properties, the storage and computational complexity can be significantly reduced, so that the efficiency of the RRCG method is greatly improved and it is now capable of dealing with much larger system with a modest computing resource. This paper also investigates various numerical inversion schemes including the CG type and multigrid (MG) methods. It has been demonstrated that the MG is an efficient and robust numerical tool for magnetic field inversion. Not only the MG produces a rapid convergence rate, the performance is not sensitive when applying to noisy data. Numerical simulations using synthetic data and real field data are reported to confirm the effectiveness of the MG method.

Key words. Magnetic inversion, Numerical algorithm, Toeplitz matrix, Multigrid method, Conjugate gradient method.

1. Introduction

Magnetic field survey is one of the most popular geophysical techniques for fast mapping of large areas in geophysical and environmental study. The survey consists of mapping one or more components of the earth geomagnetic field in order to analyze the magnetic anomalies. The magnetic anomalies mapping can be generally used as a tool to many geological applications such as estimating the basement topography, assessing the depth in oil exploration and the magnetic polarization in mineral prospecting.

Inversion model is closely related to the forward computation, and this is a key step in the geophysical survey. For the magnetic field inversion problem, the model can be expressed mathematically as an integral formulation. The inversion solution can be obtained by solving a resulting system of linear equations $Au = b$, where b is the observation magnetic field data. The major challenge of the inversion problem is due to the fact that the matrix A is often large, dense and ill-conditioned [13], thus inverting the system by a direct method is not practical and employing an numerical iterative scheme will require enormous computing resource.

Although some works have been reported for a 3-D magnetic field inversion [18, 20], in many cases, 2-D model is more preferred, this is particularly true for the aero-magnetic survey. The simplicity of the 2-D model also make the 2-D inversion practical and efficient. The inversion for tabular magnetic anomalies or thin layer magnetic anomalies have been investigated in [15, 21, 2, 22, 26].

For the irregular raw data, it can be rewritten into a uniform data conveniently by the use of a regridding procedure. Many efficient methods have been developed, for instance, Briggs [4] proposed a minimum curvature method to regrid non-uniform

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data. Cordell and Blakely [12, 3] presented an equivalent layer method (ELM), in which a fictitious source layer is introduced and the non-uniform data points are interpolated on the uniform grid. The advantage of implementing the ELM has been reported by Cooper [11], and a comparative study of ELM and the minimum curvature method can be found in [19].

The most important feature for considering a uniform field data is that the resulting coefficient matrix for the inversion problem is a symmetric Block-Toeplitz Toeplitz-Block (BTTB) matrix. Consequently, efficient and accurate numerical inversion algorithms can be developed, and the matrix-vector product can be implemented efficiently using a Fast Fourier Transform (FFT). In some other work, the BTTB structure has already been noticed. Rauth and Strohmer [23] investigated the potential field gridding problem by interpolating the non-uniform field data into a uniform field data, where the trigonometric polynomial is used to approximate the magnetic field, and the coefficients of the polynomial are computed by solving a BTTB system. However, in our work, the BTTB structure is derived directly from magnetic forward formulation. To the best of our knowledge, no systematic investigation on the BTTB structure and construction of numerical schemes utilizing the special BTTB properties in magnetic inversion problem has been carried out.

In this paper, we investigate numerical schemes for magnetic data inversion. Particular attention is focused on incorporating the BTTB structure to develop efficient numerical inversion algorithms based on the conjugate gradient (CG) type methods and the multigrid (MG) technique. The CG type methods include the standard CG (CG), preconditioned CG (PCG) and the re-weighted regularized CG (RRCG) method. A comparative study of the MG and CG type methods is presented, and the performance of these methods is validated by numerical simulations applied to the synthetic field data and the real geophysical data.

2. Forward Model

Assuming that the magnetic data covers an area which is filled with a set of vertical prisms with arbitrary horizontal section and the bottom at infinity, the magnetic anomaly reduced to the pole is given by a layer of poles on the top of each prism as shown in Figure 1.

The magnetization is defined as the magnetic moment ($\mathbf{M}$) per volume as

$$\boldsymbol{J} = \frac{d\mathbf{M}}{dv}, \tag{1}$$

which is induced by the earth magnetic field, and is the source of the magnetic anomaly. To determine the magnetic field generated by the magnetization, the concept of magnetic scalar potential ψ is introduced. When there is no free current, the magnetic scalar potential can be used to determine the magnetic H-field especially for the permanent magnets in the following way,

$$\mathbf{H} = -\nabla\psi. \tag{2}$$

It is known that the magnetic potential generated by $d\mathbf{M}$ at an arbitrary point P is defined by $d\psi = \frac{d\mathbf{M}\cdot\mathbf{r}}{\rho}$, where $\mathbf{r}$ is a coordinate of P, and ρ is the distance from P to dv. According to (1),

$$d\psi = -\left[\mathbf{J}\cdot\nabla(\frac{1}{\rho})\right] dv. \tag{3}$$