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The Zero-Mach Limit of a Class of Combustion Flow

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Abstract. The aim of this paper is to prove that compressible combustion flow in two and three space dimensions converges to the incompressible flow in the limit as the Mach number tends to zero.

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Key Words: Zero-Mach limit; combustion equation; compressible; incompressible.

1 Introduction

In this paper, we consider the equation for a compressible, isentropic or isothermal combustion flow as follows:

$$\begin{cases} \rho_t+\mathrm{div}(\rho u)=0, \\ (\rho u^j)_t+\mathrm{div}(\rho u^j u)+\tilde{\pi}_{x_j}=\varepsilon\Delta u^j, \\ (\rho z)_t+\mathrm{div}(\rho z u)=-\overline{k}(\rho,z)\rho z, \end{cases} \tag{1.1}$$

with initial value

$$\big(\rho,u,z\big)(x,t)|_{t=0}=\big(\rho_0(x),u_0(x),z_0(x)\big). \tag{1.2}$$

Here, $j=1,2\cdots n$, $n=2$ or 3; $\varepsilon>0$ is a viscosity constant; ρ, $u=(u^1,\cdots,u^n)$ and z are the fluid density, velocity and the per centum of the responseless gas in the gas flow, which are the unknown functions of $x\in R^n$, and $t\in[0,\infty)$; $\tilde{\pi}\equiv A(z)\rho^\gamma$ is the pressure, $\gamma>1$; $A=A(\cdot)$ satisfies

$$\begin{cases} A(z)\in W^{2,\infty}(R), \\ A,A_z>0, \end{cases} \tag{1.3}$$

and $\overline{k}$ satisfies

$$\begin{cases} \overline{k}=\overline{k}(\cdot,\cdot)\in W^{1,\infty}(R\times R), \\ \overline{k}\geq 0. \end{cases} \tag{1.4}$$

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We can rewrite (1.1) in dimensionless form by taking

$$x = L\overline{x}, \quad t = T\overline{t}, \quad \rho = R\overline{\rho}, \quad u = T\overline{u}/L, \quad z = T^2\overline{z}/L^2, \quad \overline{k}(\rho,z) = T\overline{\overline{k}}(R\overline{\rho},T^2\overline{z}/L^2),$$

where L, T and R are typical values of x, t and ρ, and the quantities with bars are dimensionless. Making these substitutions into (1.1), $\pi = R^{\gamma-1}A(\delta^2\overline{z})\overline{\rho}^\gamma$, $\gamma \geq 1$, then dropping the bars, we find that, for new, dimensionless variables x, t, ρ, z, u, and a new dimensionless viscosity constant ε

$$\begin{cases} \rho_t + \text{div}(\rho u) = 0, \\ (\rho u^j)_t + \text{div}(\rho u^j u) + \delta^{-2}\pi_{x_j} = \varepsilon \Delta u^j, \\ (\rho z)_t + \text{div}(\rho z u) = -\overline{k}(R\rho,\delta^2 z)\rho z, \end{cases} \tag{1.5}$$

with initial value

$$(\rho,u,z)(x,t)|_{t=0} = (\rho_0(x),u_0(x),z_0(x)). \tag{1.6}$$

We assume

$$0 \leq z_0(x) \leq 1,$$

where now δ is the Mach number, given by

$$\delta^2 = (T/L)^2,$$

δ is thus the ratio of the typical particle speed to the typical sound speed, and is therefore small, by definition, for highly subsonic flows.

We give the incompressible combustion flow:

$$\begin{cases} \text{div} w = 0, \\ \widetilde{\rho}(w^j_t + \nabla w^j \cdot w) + R^{\gamma-1}\widetilde{\rho}^\gamma A_z(0) z_{1x_j} + R^{\gamma-1}\gamma\widetilde{\rho}^{\gamma-1}A(0)\rho_{1x_j} = \varepsilon \Delta w^j, \\ z_{1t} + \nabla z_1 \cdot w = -\overline{k}(R\widetilde{\rho},0) z_1, \end{cases} \tag{1.7}$$

with initial value:

$$(w,z_1)(x,t)|_{t=0} = (w_0(x),z_{1_0}(x)). \tag{1.8}$$

In (1.7), $\widetilde{\rho}$ is a constant representing the density of the incompressible combustion flow; w, $R^{\gamma-1}\widetilde{\rho}^\gamma A_z(0) z_1 + R^{\gamma-1}\gamma\widetilde{\rho}^{\gamma-1}A(0)\rho_1$ and z_1 are respectively the velocity, pressure and per centum of the responseless gas in an incompressible flow.

Assume that $\widetilde{\rho} > 0$ and that the initial value $(w_0(x),z_{1_0}(x))$ of (1.7) satisfies

$$w_0(x) \in W^{2,2}(R^n), \quad 0 \leq z_{1_0}(x) \leq 1 \in W^{1,2}(R^n). \tag{1.9}$$

Then problem (1.7)-(1.8) has a corresponding solution (w,ρ_1,z_1) defined on $R^n \times I$ (see [1]) satisfying the following conditions:

$$\begin{cases} \rho_1 + z_1 \in L^\infty(I;L^\infty \cap W^{1,2}) \cap L^2(I;L^\infty), \\ \rho_{1t} + z_{1t}\ ,\ \nabla\rho_1 + \nabla z_1 \in (L^1 \cap L^2)(I;L^\infty) \cap L^2(I;L^2), \\ w \in L^\infty(I;W^{2,2} \cap W^{1,\infty}), \\ \nabla w, D^2_x w \in L^2(I;L^2 \cap L^\infty), \\ \rho_1 \triangle w \in L^1(I;L^2 \cap L^\infty). \end{cases} \tag{1.10}$$