Commun. Comput. Phys.
doi: 10.4208/cicp.020709.211209a

Vol. 8, No. 5, pp. 947-975
November 2010

Nonlinear Stochastic Galerkin and Collocation Methods: Application to a Ferromagnetic Cylinder Rotating at High Speed

Eveline Rosseel[1], Herbert De Gersem[2] and Stefan Vandewalle[1,*]

[1] *Computer Science Department, K. U. Leuven, Celestijnenlaan 200A, 3001 Leuven, Belgium.*
[2] *Faculty of Sciences, K. U. Leuven, Etienne Sabbelaan 53, 8500 Kortrijk, Belgium.*

Received 2 July 2009; Accepted (in revised version) 21 December 2009

Abstract. The stochastic Galerkin and stochastic collocation method are two state-of-the-art methods for solving partial differential equations (PDE) containing random coefficients. While the latter method, which is based on sampling, can straightforwardly be applied to nonlinear stochastic PDEs, this is nontrivial for the stochastic Galerkin method and approximations are required. In this paper, both methods are used for constructing high-order solutions of a nonlinear stochastic PDE representing the magnetic vector potential in a ferromagnetic rotating cylinder. This model can be used for designing solid-rotor induction machines in various machining tools. A precise design requires to take ferromagnetic saturation effects into account and uncertainty on the nonlinear magnetic material properties. Implementation issues of the stochastic Galerkin method are addressed and a numerical comparison of the computational cost and accuracy of both methods is performed. The stochastic Galerkin method requires in general less stochastic unknowns than the stochastic collocation approach to reach a certain level of accuracy, however at a higher computational cost.

AMS subject classifications: 35K60, 65N35, 65C50, 78M25

Key words: Nonlinear PDE with random coefficients, polynomial chaos, stochastic collocation method, stochastic Galerkin method, electromagnetics.

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1 Introduction

Ferromagnetic cylinders rotating at high speeds can be found as part of solid-rotor induction machines in various machining tools and in magnetic brakes. At high speeds, when the surface layer of a ferromagnetic rotor gets fully saturated, solid-rotor induction

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[*]Corresponding author. *Email addresses:* `Eveline.Rosseel@cs.kuleuven.be` (E. Rosseel), `Herbert.DeGersem@kuleuven-kortrijk.be` (H. De Gersem), `Stefan.Vandewalle@cs.kuleuven.be` (S. Vandewalle)

machines and magnetic brakes produce a higher torque [7]. As a consequence, designing solid-rotor devices with high-speed conductive parts requires to take ferromagnetic saturation effects into account. These nonlinear material properties can typically not be quantified exactly. A reliable design needs to deal with uncertainty. One of the goals of this paper is to determine to what extent uncertainty on the magnetic material parameters influences the machine properties. We will express this uncertainty by introducing stochastic variables into the mathematical model, which will take the form of a nonlinear stochastic partial differential equation.

A standard tool for solving stochastic partial differential equations is the Monte Carlo simulation method [11]. Recently also other techniques have been developed that enable to compute high-order accurate stochastic solutions and that try to reduce the large computational cost of Monte Carlo simulations. Amongst these, the stochastic Galerkin finite element method [3,14,39] and the stochastic collocation method [1,20,36] turn out to be very successful [22,35]. Similar to Monte Carlo, the stochastic collocation method is based on sampling. The multidimensional samples, called collocation points, are chosen in order to obtain an exponential convergence rate [1]. The stochastic collocation approach suffers from a curse of dimensionality for problems with many random variables. This problem can be alleviated by a so-called 'sparse grid' construction of collocation points [24,36]. The stochastic Galerkin finite element method, on the other hand, applies spectral finite element theory to convert a stochastic PDE into a set of coupled deterministic PDEs. The number of deterministic PDEs is in general smaller than for the stochastic collocation method for a same level of accuracy. Yet, the coupling of the deterministic PDEs leads to high-dimensional algebraic systems, which are expensive to solve.

In this paper, we will point out how the high-order stochastic Galerkin and collocation techniques can be applied to a nonlinear stochastic model of a ferromagnetic rotor. We consider randomness on the magnetic material properties, and allow variability on the conductivity and on the boundary conditions. The application of the stochastic collocation method to a nonlinear stochastic PDE is fairly straightforward, as it reuses deterministic simulation code. Applying a stochastic Galerkin method to a nonlinear stochastic problem, however, is nontrivial. Only few results on the stochastic Galerkin method applied to a nonlinear stochastic problem are available in the literature [19–22], and none of them treat this particular type of nonlinear convection-diffusion problem. We address the numerical and implementation issues that arise in applying the stochastic Galerkin method to the specified nonlinear model. Further, the question of which method, stochastic Galerkin or stochastic collocation, yields the most accurate solution in the lowest computational time, remains open. We shall compare the accuracy and computational cost of both stochastic solution methods through various numerical experiments. Comparative studies between the performance of the stochastic collocation and Galerkin method are rare and mostly limited to linear PDEs [5,20,35].

This paper is organized as follows. In Section 2, the mathematical model needed to describe a magnetic field in a ferromagnetic rotating cylinder, is presented. Uncertainty on the material parameters is formulated by introducing random variables into