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An hp Adaptive Uniaxial Perfectly Matched Layer Method for Helmholtz Scattering Problems

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Abstract. We propose an adaptive strategy for solving high frequency Helmholtz scattering problems. The method is based on the uniaxial PML method to truncate the scattering problem which is defined in the unbounded domain into the bounded domain. The parameters in the uniaxial PML method are determined by sharp a posteriori error estimates developed by Chen and Wu [8]. An hp-adaptive finite element strategy is proposed to solve the uniaxial PML equation. Numerical experiments are included which indicate the desirable exponential decay property of the error.

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Key words: Adaptivity, uniaxial perfectly matched layer, time-harmonic scattering, Helmholtz equation, hp-finite element method.

1 Introduction

To numerically solve the scattering problem in an unbounded domain, the first problem to be settled is to truncate the computational domain without introducing excessive error into the computed solution. Two basic approaches have been developed for this goal, the perfectly matched layer (PML) and radiation boundary conditions, of which we choose the former one to formulate our problem.

Since Berenger [4] which proposed a PML technique for solving the time dependent Maxwell equations, various constructions of PML absorbing layers have been proposed and studied in the literature (cf., e.g., Turkel and Yefet [18], Teixeira and Chew [17]). The

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basic idea of the PML technique is to surround the computational domain by a layer of finite thickness with specially designed model medium that would attenuate all the waves that propagate from inside the computational domain.

As the PML has to be truncated at a finite distance from the domain of interest, its external boundary produces artificial reflections. It is proved in Hohage et al. [13], Lassas and Somersalo [14] that the PML solution converges exponentially to the solution of the original scattering problem as the thickness of the PML layer tends to infinity. In practical applications involving PML techniques, one cannot afford to use a very thick PML layer if uniform finite element meshes are used due to excessive computational costs. On the other hand, a thin PML layer requires a rapid variation of the artificial material property which deteriorates the accuracy if too coarse mesh is used in the PML layer.

The adaptive PML technique was proposed in Chen and Wu [5] for a scattering problem by periodic structures (the grating problem), in Chen and Liu [6] for the acoustic scattering problem, and in Chen and Chen [7] for the Maxwell scattering problem. The main idea of the adaptive PML technique is to use the a posteriori error estimate to determine the PML parameters and to use the adaptive finite element method to solve the PML equations. The adaptive PML technique provides a complete numerical strategy to solve the scattering problems in the framework of finite element which produces automatically a coarse mesh size away from the fixed domain and thus makes the total computational costs insensitive to the thickness of the PML absorbing layer.

The uniaxial PML method is widely used in the engineering literature. It provides greater flexibility and efficiency to solve problems involving anisotropic scatterers as opposing to circular PML method. In Chen and Wu [8], the adaptive PML technique developed for circular PML methods in [5–7] is extended to the uniaxial PML methods and the convergence proof of the uniaxial PML method is given.

While the low-order adaptive method based on the uniaxial PML technique can save considerable computational costs, it still suffers from the drawback that the CPU time and memory storage grow rapidly as the accuracy requirement of the computed solution increases. Even for scatterers only a few wavelengths in size, the computer resources required may be excessive for computing far field patterns to a few digits of accuracy by low-order methods. One remedy is to use the hp version of the finite element which simultaneously refines the mesh and increases the degrees of elements uniformly or selectively. In contrast to the classical h version with low-order basis functions which yields algebraic decay of error in terms of the number of unknowns, the hp version enjoys the attractive feature of exponential rate of convergence with proper choices of meshes and element degrees (see, e.g., Babuška and Guo [2] for elliptic equations).

Our purpose in this paper is to recover this essential feature to achieve high accuracy of the hp FEM in the scattering problem with singular solutions, especially, for high wave numbers. Our hp-adaptive finite element strategy is based on the a posteriori error estimate developed by following the procedure of [8], of which the new ingredient is the dependence of the element degree in our local error estimator. We borrow the hp Clément interpolant in Melenk and Wohlmuth [10] to achieve this sharp hp error estimates.