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A Multiple Temperature Kinetic Model and its Application to Near Continuum Flows†

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Abstract. In an early approach, we proposed a kinetic model with multiple translational temperature [K. Xu, H. Liu and J. Jiang, Phys. Fluids **19**, 016101 (2007)]. Based on this model, the stress strain relationship in the Navier-Stokes (NS) equations is replaced by the translational temperature relaxation terms. The kinetic model has been successfully used in both continuum and near continuum flow computations. In this paper, we will further validate the multiple translational temperature kinetic model to flow problems in multiple dimensions. First, a generalized boundary condition incorporating the physics of Knudsen layer will be introduced into the model. Second, the direct particle collision with the wall will be considered as well for the further modification of particle collision time, subsequently a new effective viscosity coefficient will be defined. In order to apply the kinetic model to near continuum flow computations, the gas-kinetic scheme will be constructed. The first example is the pressure-driven Poiseuille flow at Knudsen number 0.1, where the anomalous phenomena between the results of the NS equations and the Direct Simulation Monte Carlo (DSMC) method will be resolved through the multiple temperature model. The so-called Burnett-order effects can be captured as well by algebraic temperature relaxation terms. Another test case is the force-driven Poiseuille flow at various Knudsen numbers. With the effective viscosity approach and the generalized second-order slip boundary condition, the Knudsen minimum can be accurately obtained. The current study indicates that it is useful to use multiple temperature concept to model the non-equilibrium state in near continuum flow limit. In the continuum flow regime, the multiple temperature model will automatically recover the single temperature NS equations due to the efficient energy exchange in different directions.

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Key words: Navier-Stokes equations, gas-kinetic model, non-equilibrium flows.

†Dedicated to Professor Xiantu He on the occasion of his 70th birthday.
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1 Introduction

The transport phenomena, i.e., mass, heat, and momentum transfer, in different flow regime is of a great scientific and practical interest. The classification of various flow regimes is based on the dimensionless parameter, i.e., the Knudsen number, which is a measure of the degree of rarefaction of the medium. The Knudsen number Kn is defined as the ratio of the mean free path to a characteristic length scale of the system. In the continuum flow regime, i.e., $Kn < 0.001$, the Navier-Stokes equations with linear relations between stress and strain and the Fourier's law for heat conduction are adequate to model the fluid behavior. For flows in the continuum-transition regime ($0.1 < Kn < 1$), the Navier-Stokes equations are known to be inadequate. This regime is important for many practical engineering problems, such as the simulation of microscale flows [10] and hypersonic flow around space vehicles in low earth orbit [9]. Hence, there is a strong desire and requirement for accurate models which give reliable solutions with lower computational costs.

Currently, the DSMC method is the most successful technique in the numerical prediction of low density flows [3]. However, in the continuum-transition regime, especially for the micro-channel flows, the DSMC suffers from statistical noise in the bulk velocity because of the random molecular motion. When the bulk velocity is much slower than the thermal velocity, many independent samples are needed to eliminate the statistical scattering, as for the micro-electro-mechanical system (MEMS) simulation. Alternatively, many macroscopic continuum model have been intensively developed in the literature, which include the Burnett and super-Burnett equations [4, 18], Grad's 13 moment equations [5], the regularized 13 equations [14], and many others. For high-order equations, besides the difficulties in constructing the boundary condition, another assumption is that any non-equilibrium state is only a certain perturbation of the equilibrium one. In reality, the non-equilibrium state in the near continuum flow regime may not be able to be recovered from a simple truncated expansion of an equilibrium state.

In [21], based on the gas-kinetic Bhatnagar-Gross-Krook (BGK) equation, a kinetic model with multiple translational temperature for the continuum and near continuum flow simulation was proposed. In this approach, the energy exchanges among x-, y-, and z-directions are modeled through the particle collision. Based on the kinetic model, the viscous term in the Navier-Stokes equations is replaced by the temperature relaxation in the extended NS formulations. In the continuum flow regime, the standard Navier-Stokes solutions are precisely recovered. The numerical results presented in [21] are in good agreement with the DSMC data for a wide range of Kn numbers. The anomalous temperature minimum phenomena in the force-driven Poiseuille flow case at $Kn = 0.1$ between the Navier-Stokes solutions and the DSMC results are well captured by the multiple temperature kinetic model. The current study is to further develop the multiple temperature model by the following. First, in order to incorporate the flow physics near the wall, such as the flow behavior inside the Knudsen layer, the particle-particle collision as well as particle-wall collisions will be included in the current model through the modifi-