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The Error-Minimization-Based Strategy for Moving Mesh Methods

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Abstract. The typical elements in a numerical simulation of fluid flow using moving meshes are a time integration scheme, a rezone method in which a new mesh is defined, and a remapping (conservative interpolation) in which a solution is transferred to the new mesh. The objective of the rezone method is to move the computational mesh to improve the robustness, accuracy and eventually efficiency of the simulation. In this paper, we consider the one-dimensional viscous Burgers' equation and describe a new rezone strategy which minimizes the L_2 norm of error and maintains mesh smoothness. The efficiency of the proposed method is demonstrated with numerical examples.

Key words: Moving meshes; Burgers' equation; error estimates.

1 Introduction

In a numerical simulation of fluid flow, the relationship of the motion of computational mesh to the motion of the fluid is an important issue. There are two choices that are typically made. In Lagrangian methods the mesh moves with the local fluid velocity, while in Eulerian methods the fluid flows through the mesh that is fixed in space.

In general, the motion of the mesh can be chosen arbitrarily. The moving mesh method described in this paper exploits this freedom to improve the robustness, accuracy and eventually efficiency of the simulation. The main elements in the simulation with arbitrary moving meshes are an explicit time integration scheme, a rezone method in which a new mesh is defined, and a remapping (conservative interpolation) in which a solution is transferred to the new mesh [44].

Our ultimate goal is to develop a robust and efficient rezone strategy for the system of gasdynamics equations in 2D and 3D. Since this is the very challenging task, we commence with a simpler problem: the one-dimensional viscous Burgers' equation. This equation has many

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important features of gasdynamics equations. It expresses conservation law and its solution can develop shock-like structures.

In this paper, we consider both the Lagrangian and Eulerian forms of Burgers' equation. The new rezone method employs the *look ahead strategy*. It changes the mesh at time t^n in such a way to minimize the L_2-norm of error at time t^{n+1}. The analysis shows that under reasonable assumptions about mesh smoothness, solution regularity, accuracy of the remapping and time step, the leading term in the error depends on accuracy with which the solution at time t^n is represented by its exact mean values. The latter is the well-known interpolation problem of the best piecewise constant fit with adjustable nodes [6]. The remapping is based on the linearity preserving methods from [35] which are second-order accurate for viscous Burgers' equation.

The error analysis for viscous Burgers' equation assumes that the mesh is smooth. The mesh smoothness is absolutely critical for stability of the overall simulation. Therefore, we modified the standard algorithm for the best piecewise constant fit to guarantee smoothness of the resulting mesh and still reduce the error. We found that the modified algorithm gives just slightly larger error in comparison with the original algorithm.

The paper outline is as follows. In Section 2, we describe the Eulerian and the Lagrangian forms of viscous Burgers' equation. In Section 3, we describe the moving mesh method with the error-minimization-based (EMB) rezone strategy for both the Eulerian and the Lagrangian forms of Burger's equation. In Section 4, we analyze the moving mesh method. In Section 5, we present results of numerical experiments. In Section 6, we give a short overview of related methods, emphasize some common ideas and important distinctions with our method. In concluding Section 7, we discuss plans for future work.

2 Viscous Burgers' equation

In this section we consider different forms and some properties of Burgers' equation that will be useful for the purpose of this paper.

2.1 Burgers' equation in the Eulerian form

The standard Eulerian form of the one-dimensional Burgers' equation is

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \varepsilon\frac{\partial^2 u}{\partial x^2}, \qquad t \in (0,T), \tag{2.1}$$

where $\varepsilon << 1$ is a given constant. For a finite ε, solution of equation (2.1) is a smooth function which may have sharp gradients whose steepness depends on how small ε is. Equation (2.1) is subject to the initial condition

$$u(x,0) = U(x), \qquad x \in (-\infty,+\infty). \tag{2.2}$$

For simplicity, in this section, we assume that

$$u(x,t), \quad \frac{\partial u}{\partial x}(x,t) \to 0 \quad \text{when} \quad x \to \pm\infty\,.$$