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The Levels-Recursive Algorithm for Vector Valued Interpolants by Triple Branched Continued Fractions†

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Abstract. A kind of triple branched continued fractions is defined by making use of Samelson inverse and Thiele-type partial inverted differences [1]. In this paper, a levels-recursive algorithm is constructed and a numerical example is given.

Key words: Vector valued interpolant; levels-recurrence algorithm algorithm; triple branched continued fractions.

AMS subject classifications: 41A20, 65D05

1 Introduction

Let $\Pi^{l,m,n}$ be a set of points in three dimensional space $\mathbf{R}^3$,

$$\Pi^{l,m,n} = \{(x_i, y_j, z_k),\ i = 0, 1, \cdots l;\ j = 0, 1, \cdots m;\ k = 0, 1, \cdots n\}.$$

Let a $d-$dimensional vector $\mathbf{v}_{i,j,k}$ be given at every point $(x_i, y_j, z_k) \in \Pi^{l,m,n}$ and let $\mathbf{V}^{l,m,n}$ denote the collection of all those vectors. For a given complex vector $\mathbf{v} \in \mathbf{C}^d$, its generalized inverse (or the Samelson inverse) is defined as

$$\mathbf{v}^{-1} = \frac{\mathbf{v}^*}{|\mathbf{v}|^2},$$

where $\mathbf{v}^*$, as usual, denotes the complex conjugate of $\mathbf{v}$.

In the paper [1], Tan-Tang constructed a triple branched continued fractions by means of well-defined Thiele-type partial difference. In the paper [2], Tan-Tang gave a recursive algorithm for a kind of triple branched continued fractions. In this paper, a levels-recursive algorithm is constructed by so-called backward recurrence relation for vector valued continued fraction [4].

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2 The levels-recurrence algorithm

We first describe the levels-recurrence algorithm, and then provide an interpolation property relevant to the algorithm.

Step 1. Initialization: For $j = 0, 1, \cdots, m$; $k = 0, 1, \cdots, n$, let

$$\mathbf{a}_0(y_j, z_k) = \mathbf{v}_{0,j,k} \tag{1}$$

Step 2. Recurrence in x direction. For $i = 1, \cdots, l$; $j = 0, 1, \cdots, m$; $k = 0, 1, \cdots, n$;
1) Let

$$\mathbf{P}^{0,0,0}_{i,j,k} = \mathbf{v}_{i,j,k} - \mathbf{a}_0(y_j, z_k), \quad Q^{0,0,0}_{i,j,k} = 1, \quad Q^{1,0,0}_{i,j,k} = |\mathbf{v}_{i,j,k} - \mathbf{a}_0(y_j, z_k)|^2. \tag{2}$$

2) For $u = 1, 2, \cdots, i-1$; $i = 2, 3, \cdots, l$; let

$$\mathbf{P}^{u,0,0}_{i,j,k} = -\mathbf{a}_u(y_j, z_k) Q^{u,0,0}_{i,j,k} + (x_i - x_{u-1}) \mathbf{P}^{u-1,0,0}_{i,j,k}. \tag{3}$$

3) For $u = 2, 3, \cdots, i-1$; $i = 3, \cdots, l$; let

$$Q^{u,0,0}_{i,j,k} = |\mathbf{a}_{u-1}(y_j, z_k)|^2 Q^{u-1,0,0}_{i,j,k} + (x_i - x_{u-2})^2 Q^{u-2,0,0}_{i,j,k} - 2(x_i - x_{u-2}) \mathbf{a}_{u-1}(y_j, z_k) \mathbf{P}^{u-2,0,0}_{i,j,k}. \tag{4}$$

4) Let

$$\mathbf{a}_i(y_j, z_k) = (x_i - x_{i-1}) Q^{i-1,0,0}_{i,j,k} / \mathbf{P}^{i-1,0,0}_{i,j,k}. \tag{5}$$

Step 3. Recurrence in y direction. For $j = 1, 2, \cdots, m; k = 0, 1, \cdots, n$;
1) For $p = 0, 1, \cdots, l$, let

$$\mathbf{b}_{p,0}(z_k) = \mathbf{a}_p(y_0, z_k). \tag{6}$$

2) Let $\mathbf{P}^{0,0}_{j,k} = -\mathbf{a}_p(y_0, z_k) + \mathbf{a}_p(y_j, z_k)$ and

$$Q^{0,0}_{j,k} = 1, \qquad Q^{0,1}_{j,k} = |-\mathbf{a}_p(y_0, z_k) + \mathbf{a}_p(y_j, z_k)|^2. \tag{7}$$

3) For $v = 1, 2, \cdots, j-1$; $j = 2, \cdots, m$; let

$$\mathbf{P}^{0,v}_{j,k} = -\mathbf{b}_{p,v}(z_k) Q^{0,v}_{j,k} + (y_j - y_{v-1}) \mathbf{P}^{0,v-1}_{j,k}. \tag{8}$$

4) For $v = 2, \cdots, j-1$; $j = 3, \cdots, m$;

$$Q^{0,v}_{j,k} = |\mathbf{b}_{p,v-1}(z_k)|^2 Q^{0,v-1}_{j,k} + (y_j - y_{v-2})^2 Q^{0,v-2}_{j,k} - 2(y_j - y_{v-2}) \mathbf{b}_{p,v-2}(z_k) \mathbf{P}^{0,v-2}_{jk}. \tag{9}$$

5) Let

$$\mathbf{b}_{p,j}(z_k) = (y_j - y_{j-1}) / \mathbf{P}^{0,j-1}_{j,k} Q^{0,j-1}_{i,k}. \tag{10}$$

Step 4. Recurrence in z direction; For $i = 1, \cdots, l$; $q = 0, 1, \cdots, m$; $k = 0, 1, \cdots, n$.
1) Let

$$\mathbf{c}_{p,q,0} = \mathbf{b}_{p,q}(z_0). \tag{11}$$

2) Let

$$\mathbf{P}^0_k = -\mathbf{c}_{p,q,0} + \mathbf{b}_{p,q}(z_k), Q^0_k = 1, Q^1_{k+1} = |-\mathbf{c}_{p,q,0} + \mathbf{b}_{p,q,0}|^2. \tag{12}$$

3) For $r = 1, 2, \cdots, k-1$; $k = 2, \cdots, n$, let

$$\mathbf{P}^r_k = -\mathbf{c}_{p,q,r} Q^r_k + (z_k - z_{r-1}) \mathbf{P}^{r-1}_k. \tag{13}$$