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STEP-LIKE CONTRAST STRUCTURE OF SINGULARLY PERTURBED OPTIMAL CONTROL PROBLEM*

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Abstract

The existence of step-like contrast structure for a class of singularly perturbed optimal control problem is presented by contrast structure theory. By means of direct scheme of boundary function method, we construct the uniformly valid asymptotic solution for the singularly perturbed optimal control problem. As an application, an example is given to illustrate the main result in this paper.

Mathematics subject classification: 34B15, 34E15.
Key words: Singular perturbation, Optimal control problem, Contrast structure.

1. Introduction

The problem of contrast structure is a singularly perturbed problem whose solutions with both internal transition layers and boundary layers (see, e.g., [1-3]). The significant feature of the solution is that it will vary rapidly in the thin internal layer. The contrast structure has a strong application background. For example, in the study of physics , there are cases that their solutions vary rapidly in the interior of domain. In recent years, the study of contrast structure is one of the hot research topics in the study of singular perturbation theory. More and more scholars begin to pay attention to the contrast structure of variational problem. In [4], [5], the authors consider the contrast structures for the simplest vector variational problem and scalar variational problem. One of the basic difficulties for such a problem is unknown of where an internal transition layer is in advance.

Currently, there are mainly two ways to solve this problem. The first way is through the boundary function method [6]. Usually, this method is applied to necessary or sufficient optimality conditions. The second alternative is through direct scheme of boundary function method, which consists in a direct expansion of the optimal control problem. we will apply the direct scheme to the singularly perturbed optimal control problem. As a result of the scheme, we get a minimizing control sequence, each new control approximation decreases the performance index of the given problem. It should be noted that the direct scheme not only make it easy to obtain the relations for the high-order approximations, but also show the nature of the optimal control problem.

In this present paper, we not only prove the existence of step-like contrast structure for the singularly perturbed optimal control problem, but also construct asymptotic solution to the optimal controller and optimal trajectory.

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2. Problem Formulation

Consider the singularly perturbed optimal control problem

$$\begin{cases} J[u] = \int_0^T f(y,u,t)\,\mathrm{d}t \to \min\limits_{u}, \\ \mu \dfrac{dy}{dt} = a(t)y + b(t)u, \\ y(0,\mu) = y^0\,, \quad y(T,\mu) = y^T. \end{cases} \tag{2.1}$$

where $\mu > 0$ is a small parameter. The following assumptions are fundamental in the theory for the problem in question.

$A_1.$ Suppose that the function $f(y,u,t)$ is sufficiently smooth on the domain $D = \{(y,u,t)| \mid y \mid < A, u \in R, 0 \le t \le T\}$, where A is positive constant.
$A_2.$ Suppose that $f_{uu}(y,u,t) > 0$ on the domain D.

Formally setting $\mu = 0$ in (2.1), we obtain the reduced problem

$$J[\bar{u}] = \int_0^T f(\bar{y},\bar{u},t)\,\mathrm{d}t \to \min_{\bar{u}}, \quad \bar{u} = -b^{-1}(t)a(t)\bar{y}. \tag{2.2}$$

For convenience, problem (2.2) can be written in the following equivalent form

$$J[\bar{u}] = \int_0^T F(\bar{y},t)\mathrm{d}t \to \min_{\bar{y}},$$

where $F(\bar{y},t) = f(\bar{y}, -b^{-1}(t)a(t)\bar{y}, t)$.

$A_3.$ Suppose that there exist two isolated functions $\bar{y} = \varphi_1(t)$, $\bar{y} = \varphi_2(t)$ such that

$$\min_{\bar{y}} F(\bar{y},t) = \begin{cases} F(\varphi_1(t),t) & 0 \le t \le t_0, \\ F(\varphi_2(t),t), & t_0 \le t \le T, \end{cases} \tag{2.3}$$

$$\lim_{t \to t_0^-} \varphi_1(t) \ne \lim_{t \to t_0^+} \varphi_2(t).$$

$A_4.$ Suppose that the transition point t_0 is determined by the following equation

$$F(\varphi_1(t_0),t_0) = F(\varphi_2(t_0),t_0),$$

and satisfies the condition

$$\frac{d}{dt}F(\varphi_1(t_0),t_0) \ne \frac{d}{dt}F(\varphi_2(t_0),t_0).$$

It follows from assumption A_3 that

$$\bar{u}(t) = \begin{cases} \alpha_1(t) = -b^{-1}(t)a(t)\varphi_1(t), & 0 \le t < t_0, \\ \alpha_2(t) = -b^{-1}(t)a(t)\varphi_2(t), & t_0 < t \le T, \end{cases}$$

$$\begin{cases} F_y(\varphi_1(t),t) = 0, & F_{yy}(\varphi_1(t),t) > 0, & 0 \le t \le t_0, \\ F_y(\varphi_2(t),t) = 0, & F_{yy}(\varphi_2(t),t) > 0, & t_0 \le t \le T. \end{cases} \tag{2.4}$$