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A NUMERICAL METHOD FOR THE SIMULATION OF FREE SURFACE FLOWS OF VISCOPLASTIC FLUID IN 3D*

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Abstract

In this paper we study a numerical method for the simulation of free surface flows of viscoplastic (Herschel-Bulkley) fluids. The approach is based on the level set method for capturing the free surface evolution and on locally refined and dynamically adapted octree cartesian staggered grids for the discretization of fluid and level set equations. A regularized model is applied to handle the non-differentiability of the constitutive relations. We consider an extension of the stable approximation of the Newtonian flow equations on staggered grid to approximate the viscoplastic model and level-set equations if the free boundary evolves and the mesh is dynamically refined or coarsened. The numerical method is first validated for a Newtonian case. In this case, the convergence of numerical solutions is observed towards experimental data when the mesh is refined. Further we compute several 3D viscoplastic Herschel-Bulkley fluid flows over incline planes for the dam-break problem. The qualitative comparison of numerical solutions is done versus experimental investigations. Another numerical example is given by computing the freely oscillating viscoplastic droplet, where the motion of fluid is driven by the surface tension forces. Altogether the considered techniques and algorithms (the level-set method, compact discretizations on dynamically adapted octree cartesian grids, regularization, and the surface tension forces approximation) result in efficient approach to modeling viscoplastic free-surface flows in possibly complex 3D geometries.

Mathematics subject classification: 65M06, 76D27, 76D99.
Key words: Free surface flows, Viscoplastic fluid, Adaptive mesh refinement, Octree meshes.

1. Introduction

Free surfaces flows of yield stress fluids are common in nature: lava flows, snow avalanches and debris flows, as well as in engineering applications: flows of melt metal, fresh concrete, pastes and other concentrated suspensions [3,34]. Although the rheology of such materials can be quite complicated, viscoplastic models, for example the Herschel-Bulkley model, are often used to describe the strain rate – stress tensor relationship and predict the fluids dynamics with reasonable accuracy, see, e.g., [13,24]. Modeling such phenomena numerically is a challenging

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task due to the non-trivial coupling of complex flow dynamics and free surface evolution. Substantial progress has been made during the last two decades in developing efficient and accurate numerical methods for computing flows with free surfaces and interfaces, see, e.g., [42, 43] and references therein. The level set method is an implicit surface-capturing technique [45] which was proved to be particular efficient for handling free surfaces which may undergo complex topological changes. The method is extensively used for numerical modeling of free-surface flows with finite difference [37], finite volume [22] and finite element [7, 8] methods as discretization techniques. Most of this research has been focused on application to Newtonian free-surface and interface flows.

Numerical simulations of viscoplastic fluid flow has already attracted a lot of attention, see for example the review papers [16, 19]. Yet the accurate modeling of free-surface viscoplastic fluid flows poses a serious challenge. The previous studies include the application of the Arbitrary Langrangian–Eulerian method for free-surface tracking of axisymmetric squeezing Bingham flows [27], volume of fluid surface tracking for 2D Bingham flows [2], the free interface lattice Boltzmann model [21], the simulation of viscoplastic fluids over incline planes in shallow layer approximations, see, e.g., [4, 6, 26]. The present paper develops a numerical method for simulation of complex 3D viscoplastic fluid flows based on the free surface capturing by the level set method.

The numerical methodology studied here is based on several other important ingredients, besides the level set method. To approximate complex geometries emerging in the process of the free surface evolutions we use adaptive cartesian grids dynamically refined near the free surfaces and coarsened in the fluid interior. We note that using grids adaptively refined towards the free surface is a common practice, see, e.g., [10,22]. Although much of the adaptive methods studied in the literature are based on locally refined triangulations (tetrahedra) and finite element discretizations, see, e.g., [10, 18], adaptive (octree) cartesian grids are often more convenient for frequent and routine executions of refining / coarsening procedures in the course of time integration. For the application of such grids in image processing, the visualization of amorphous medium, free surface Newtonian flow computations and other applications where non-trivial geometries occur see, e.g., [31, 33, 35, 39, 44]. We combine the mesh adaptation with a splitting algorithm for time integration. The splitting scheme decouples each time step into separate advection, plasticity, div-free correction, and level-set function update substeps. For the sake of adaptation, the grid is dynamically refined or coarsened according to the distance to the evolving free boundary on every time step. For the space discretization we use a finite difference method on octree cartesian meshes with the staggered allocation of velocity–pressure nodes. Further important ingredients of the algorithm, the preserving of the distance property of the discrete level set functions, and the approximation of the normal vectors and the curvatures of the free surface, are briefly discussed.

The remainder of the paper is organized as follows. Section 2 reviews the mathematical model. In Section 3 we discuss the details of the numerical approach: the splitting algorithm for time integration of the coupled system of the Herschel-Bulkley fluid model and the level set function equations, a finite difference method for space discretization, volume correction and reinitialization methods for the level set function. Numerical results for several 3D test problems are presented in Section 4. Numerical tests include the Newtonian broken dam problem, the viscoplastic Herschel-Bulkley fluid flow over incline planes and freely oscillating viscoplastic droplet. Section 5 contains some closing remarks.