Journal of Computational Mathematics
Vol.28, No.3, 2010, 401–417.

http://www.global-sci.org/jcm
doi:10.4208/jcm.2009.10-m2873

ACCURATE ATTENUATION CORRECTION FOR ALGEBRAIC RECONSTRUCTION TECHNIQUE IN SPECT*

Elie Nasr
IMB, Institute of Mathematics of Bordeaux, UMR 5251, Université Bordeaux 1, 351, Cours de la Libration - 33405 Talence, France
Email: elienasr@usek.edu.lb

Abstract

We present a new iterative reconstruction algorithm to improve the algebraic reconstruction technique (ART) for the Single-Photon Emission Computed Tomography. Our method is a generalization of the Kaczmarz iterative algorithm for solving linear systems of equations and introduces exact and implicit attenuation correction derived from the attenuated Radon transform operator at each step of the algorithm. The performances of the presented algorithm have been tested upon various numerical experiments in presence of both strongly non-uniform attenuation and incomplete measurements data. We also tested the ability of our algorithm to handle moderate noisy data. Simulation studies demonstrate that the proposed method has a significant improvement in the quality of reconstructed images over ART. Moreover, convergence speed was improved and stability was established, facing noisy data, once we incorporate filtration procedure in our algorithm.

Mathematics subject classification: 65F10, 65R32, 68U10, 92C55.
Key words: Single-photon emission computed tomography, Attenuated radon transform, Algebraic reconstruction technique, Attenuation correction.

1. Introduction

1.1. The medical aspect

Single-Photon Emission Computed Tomography (SPECT) is a nuclear medical imaging mechanism used to determine the concentration of some biologically active molecules in some specific zone of a human body in terms of their activity distribution. Some radiopharmaceutical product is first injected into the patient's organ, with a Gamma-emitting isotope. The radiations are trapped by a Gamma-camera with detectors which is materialized by an acquisition plane, see Fig. 1.1, rotating about the patient along a grid of specific angles. The intensity of such radiations is therefore measured in each direction orthogonal to the acquisition system plane in various angular positions.

The measured quantities of traveling photons are the exponentially weighted averages of the activity distribution along straight lines on their way from the source to the detector. The reason for the exponential weight is the presence of a position-dependent attenuation coefficient (see [12]) due to photon interaction with the organic tissue in which the radiations go through.

* Received July 21, 2008 / Revised version received September 16, 2008 / Accepted October 20, 2009 / Published online February 1, 2010 /

1.2. The mathematical model

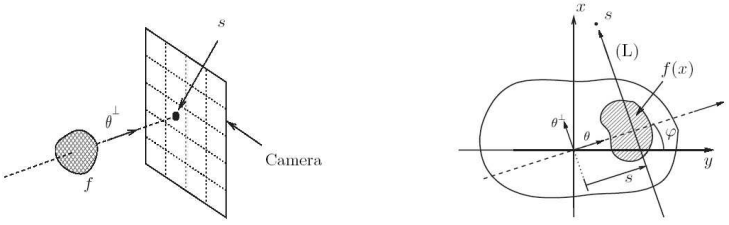
We will present here a two-dimensional model, so that the detector will be materialized by a grid of collimators regularly spaced along a line C which modelizes the Gamma-camera. Let $(x,y) \longmapsto f(x,y)$ denote the activity distribution and $(x,y) \longmapsto a(x,y)$ the position-dependent attenuation coefficient. When the detector is positioned in order to register radiations which propagate along lines L with direction $\theta^\perp$, the measured intensity $g(\theta,\ s(L))$ corresponding to the radiation along such line distant from $s = s(L)$ to the origin (the center of the exploration zone) (see Figs. 1.1a and 1.1b), is given by the attenuated Radon transform $(\theta, s) \longmapsto R_{a,\ \theta}[f](s)$ defined as :

$$\begin{aligned} R_{a,\theta}\ [f](s) &= \int_{\langle (x,y),\ \theta\rangle = s} \exp\Big(- \int_0^{+\infty} a((x,y) + t\theta^\perp)\, dt\Big)\, f(x,y)\, d\lambda_{s,\theta}(x,y) \\ &= g(\theta,\ s). \end{aligned} \tag{1.1}$$

Here $d\lambda_{s,\theta}$ stands for the restriction of the Lebesgue area measure in $\mathbb{R}^2$ to the straight line $\langle (x,y)\,,\, \theta\rangle = s$, where

$$\theta = \begin{pmatrix} \cos\varphi \\ \sin\varphi \end{pmatrix}, \quad \theta^\perp = \begin{pmatrix} -\sin\varphi \\ \cos\varphi \end{pmatrix} \quad \text{and} \quad 0 \le \theta < \varphi_{\max}.$$

The unknown activity distribution is materialized in (1.1) by f. So the problem is to recover f from the projections data $g(\theta,\ s)$, assuming of course some *a priori* information about the tissue attenuation distribution a. When a is known, there are various direct (*via* analytic formulas) or iterative techniques to recover f.



(a) (b)

Fig. 1.1. Scanner

1.3. Previous inversion methods

Direct analytic methods lie on explicit exact inversion formulas for the attenuated Radon transform. When $a \equiv 0$, the problem is solved by the classical Radon inversion formulas in dimension two (see Natterer [15] and Quinto [18] for more details about Radon's inversion formulas). When the attenuation tissue distribution a remains constant on the support of f, then the attenuated Radon transform is reduced to the exponential Radon transform, for which Tretiak and Metz [23] suggested an exact inversion formula. Nevertheless, none of these analytical methods is accurate respect to the reconstruction of f in case of an arbitrary strongly non-uniform attenuation tissue distribution, which is usually the case in any realistic SPECT problem. Novikov [17] discovered an exact analytical inversion formula for the attenuated Radon transform when a is an arbitrary tissue attenuation distribution. Another derivation of