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NUMERICAL APPROXIMATION OF TRANSCRITICAL SIMPLE BIFURCATION POINT OF THE NAVIER-STOKES EQUATIONS*[1)]

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Abstract

The extended system of nondegenerate simple bifurcation point of the Navier-Stokes equations is constructed in this paper, due to its derivative has a block lower triangular form, we design a Newton-like method, using the extended system and splitting iterative technique to compute transcritical nondegenerate simple bifurcation point, we not only reduces computational complexity, but also obtain quadratic convergence of algorithm.

Mathematics subject classification: 35A40, 65M60, 65N30, 65J15, 47H15.
Key words: Nondegenerate simple bifurcation point, Splitting iterative method, The extended system.

0. Introduction

Bifurcation problem of the Navier-Stokes equations has been studied rather extensively in the last years, see Li/Mei/Zhang(1986)[5], and M.Golubitsky/D.G.Schaefer(1988)[6], Allgower/E.Bohmer(1990) [7]. in this paper we discussed numerical approximate method of nondegenerate simple bifurcation point of the Navier-Stokes equations, the content of the paper is arranged as follows, first we introduce the Navier-Stokes equations and its operator form in the section 1, and discuss property of nondegenerate simple bifurcation points. in the section 2 we will construct a extended system as a tool for computing nondegenerate simple bifurcation points. in the section 3 we give a Newton-like method for computing transcritical nondegenerate simple bifurcation point, splitting iterative technique is used to compute transcritical nondegenerate simple bifurcation point of the Navier-Stokes equations. in the section 4 we will make numerical experiment.

1. Navier-Stokes Equation and its Nondegenerate Simple Bifurcation Point

We consider the stationary Navier-Stokes equations which has homogeneous boundary conditions

$$\begin{cases} -\nu\Delta u + (u\cdot\nabla)u + \nabla p = f, & x\in\Omega; \\ \mathrm{div} u = 0, & x\in\Omega; \\ u|_{\partial\Omega} = 0. \end{cases} \tag{1.1}$$

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Ω is a bounded and smooth domain of $R^m, m = 2, 3$, moreover $f \in [L^2(\Omega)]^m$, ν is the coefficient of kinematic viscosity.

It is well know that the uniqueness of solution of the stationary Navier-Stokes equations has only been proved under the assumptions that Reynolds number is sufficiently small, or f is sufficiently small, otherwise its solution may be not unique[1−3], for this reason it is very important to discuss efficient numerical algorithm of singular solution for Navier-Stokes equations.

Define function space

$$V = \{u \in [H_0^1(\Omega)]^m; \quad \text{div}u = 0\}$$
$$\mathcal{H} = \{u \in [L^2(\Omega)]^m; \quad \text{div}u = 0, u \cdot n|_{\partial\Omega} = 0\},$$

n denotes the outward normal vector on $\partial\Omega$. the scalar product and norm of $L^2(\Omega)^m$ are denoted by $(\cdot,\cdot)$, $|\cdot|$ on $\mathcal{H}$, Define the following scalar product on V

$$((u, v)) = (\nabla u, \nabla v), \quad \forall u, v \in V$$

$||\cdot||$ denotes its corresponding norm, variational formulation of the Navier-Stokers equations may be stated as follows [1][2]

$$\lambda a_0(u, v) + a(u, u, v) - (f, v) = 0, \quad \forall v \in V, \tag{1.2}$$

where $\lambda = \nu = Re^{-1}$ bilinear from $a_0(\cdot,\cdot)$ and trilinear form $a(\cdot,\cdot,\cdot)$ are defined by

$$a_0(u, v) = (\nabla u, \nabla v), \quad \forall u, v \in V,$$
$$a(u, v, w) = \textstyle\int_\Omega (u \cdot \nabla) v \cdot w dx, \quad \forall u, v, w \in V.$$

introduce bilinear from $B(\cdot,\cdot) : V \times V \to V'$

$$< B(u, v), w >= a(u, v, w), \quad \forall u, v, w \in V,$$

where $< \cdot,\cdot >$ denotes duality pairing on $V' \times V$. let $T(u) = \mathcal{A}^{-1}[B(u, u) - f]$, where $\mathcal{A}$ is stokes operator , then operator form of the Navier-Stokes equations can be writ as follows[1][2]

$$G(u, \lambda) := \lambda u + T(u) \tag{1.3}$$

it is Frechet differentiable and $D_uG(u, \lambda) = \lambda I + T'(u)$, it is clear that $\forall u \in V, T'(u)$ is a compact operator form V into V [1][2] , and $G : V \times R \to V$ is a nonlinear Fredholm operator with 0-index,

In the sequel the subindex 0 indicate the evaluations of function at the point (u_0, λ_0). with some calculation, we obtain:

$$D_uG_0 = \lambda_0 I + T'(u_0) = \lambda_0 I + \mathcal{A}^{-1}[B(u_0, \cdot) + B(\cdot, u_0)], \tag{1.4}$$
$$D_uG_0^* = \lambda_0 I + T'^*(u_0) = \lambda_0 I + \mathcal{A}^{-1}[B^*(u_0, \cdot) + B^*(\cdot, u_0)], \tag{1.5}$$
$$D_{uu}G_0 = T''(u_0) = \mathcal{A}^{-1}[B(\cdot,\cdot) + B(\cdot,\cdot)], \tag{1.6}$$
$$D_\lambda G_0 = u_0; \quad D_{u\lambda}G_0 = I; \quad D_{\lambda\lambda}G_0 = 0, \tag{1.7}$$

Setting ϕ, ψ are eigenfunction of D_uG_0 and $D_uG_0^*$ corresponding to 0 eigenvalue respectively, namely

$$Ker(D_uG_0) = Span\{\phi\}, \quad ||\phi|| = 1 \tag{1.8}$$
$$Ker(D_uG_0^*) = Span\{\psi\}, \quad ||\psi|| = 1 \tag{1.9}$$
$$((\phi, \psi)) = 1 \tag{1.10}$$

Fredholm theory shows that

$$\text{Range}(D_uG_0) = \{u \in V, ((u, \psi)) = 0\} \tag{1.11}$$