Journal of Computational Mathematics, Vol.16, No.3, 1998, 267–274.

A SPLITTING ITERATION METHOD FOR DOUBLE X_0-BREAKING BIFURCATION POINTS*

Rui-song Ye
(*Institute of Mathematics, Shantou University, Guangdong 515063, China*)

Abstract

A splitting iteration method is proposed to compute double X_0-breaking bifurcation points. The method will reduce the computational work and storage, it converges linearly with an adjustable speed. Numerical computation shows the effectiveness of splitting iteration method.

Key words: Double X_0-breaking bifurcation point, splitting iteration method, extended system

1. Introduction

Consider the following two-parameter dependent nonlinear problem

$$f(x,\lambda,\mu)=0,\ \ f:X\times R^2\rightarrow X, \tag{1.1}$$

where $X=R^n$, λ,μ are real parameters, $f\in C^r(r\geq 3)$, $D_xf_0(\equiv D_xf(x_0,\lambda_0,\mu_0))$ is a Fredholm map with index zero. One of our main assumptions, which arise in many applications[1,2,5–7], is that f satisfies Z_2-symmetry: there exists a linear operator $S:X\rightarrow X$ such that

$$S\neq I, S^2=I, Sf(x,\lambda,\mu)=f(Sx,\lambda,\mu),\quad \forall(x,\lambda,\mu)\in X\times R^2 \tag{1.2}$$

It is well-known that X has the following natural decomposition:

$$X=X_s\oplus X_a,$$

where

$$X_s=\{x\in X:Sx=x\},\quad X_a=\{x\in X:Sx=-x\}$$

are the set of symmetric elements and the set of anti-symmetric elements respectively[7].

We also assume that there is an invariant subspace $X_0\subset X_s$ such that

$$f(x,\lambda,\mu)\in X_0,\quad \forall(x,\lambda,\mu)\in X_0\times R^2. \tag{1.3}$$

* Received September 25, 1995.

The usual and best case is $X_0 = \{0\}$. We call (x_0, λ_0, μ_0) a singular point of (1.1) if $f(x_0, \lambda_0, \mu_0) = 0$ and $\dim(\text{Null}(f_x(x_0, \lambda_0, \mu_0)) \geq 1$. In this paper, we are concerned with double X_0-breaking bifurcation points (x_0, λ_0, μ_0) in the sense that[6]

$$f(x_0, \lambda_0, \mu_0) = 0, x_0 \in X_0, \tag{1.4a}$$

$$\text{Null } (D_x f_0) = \text{span } \{\phi_s, \phi_a\}, \phi_s \in X_s, \phi_s \notin X_0, \phi_a \in X_a, \tag{1.4b}$$

$$\text{Range } (D_x f_0) = \{y \in X : \langle \psi_s, y\rangle = \langle \psi_a, y\rangle = 0\},\ \psi_s \in X_s, \psi_a \in X_a, \tag{1.4c}$$

$$\langle \psi_s, D_\lambda f_0\rangle = \langle \psi_s, D_\mu f_0\rangle = 0. \tag{1.4d}$$

In addition, as is common, we assume that $\langle \psi_r, \phi_r\rangle = \langle \psi_r, \psi_r\rangle = \langle \phi_r, \phi_r\rangle = 1$, $r = s, a$, $\langle \psi_r, \phi_\delta\rangle = 0$, $(r, \delta) = (s, a)$ or (a, s). X_0-breaking bifurcation point is one of the three most important kinds of bifurcation points (the others are turning points and pitchfork points[2]). For the computation of double X_0-breaking bifurcation points of (1.1), Werner[6] proposed a regular extended system which is a direct method and is at least three times larger than the original equation (1.1). Here we will propose a splitting iteration method. The method produces smaller systems and it could simultaneously compute the point (x_0, λ_0, μ_0), the null vectors of $D_x f_0$, $D_x f_0{}^*$ in a coupled way. This method converges linearly with an adjustable speed and its computational cost at each iteration step remains the same level as that for the regular solution of (1.1)[3,4].

We will construct small extended systems in section 2, then propose the splitting iteration method in section 3. Numerical examples are given in section 4 to show the effectiveness of the method.

2. Extended Systems

First, we introduce the following lemma, which could be proved directly by differentiating (1.2).

Lemma 1. *$\forall x \in X_0,\ \lambda \in R, \mu \in R,$*

(i) $f(x, \lambda, \mu), D_\lambda f(x, \lambda, \mu), D_\mu f(x, \lambda, \mu) \in X_0$;

(ii) X_0, X_s and X_a are invariant subspaces of $D_x f(x, \lambda, \mu), D_{x\lambda} f(x, \lambda, \mu)$, $D_{x\mu} f(x, \lambda, \mu)$;

(iii) $\forall v, w \in X_0\ D_{xx} f(x, \lambda, \mu) vw \in X_0$;

(iv) $\forall v \in X_s, w \in X_s$, or $\forall v \in X_a, w \in X_a$, $D_{xx} f(x, \lambda, \mu) vw \in X_s$;

(v) $\forall v \in X_s, w \in X_a$, $D_{xx} f(x, \lambda, \mu) vw \in X_a$.

It follows from Lemma 1 and (1.4d) that there exist $v_0, u_0 \in X_0$ such that

$$D_x f_0 v_0 + D_\lambda f_0 = 0, \tag{2.1a}$$

$$D_x f_0 u_0 + D_\mu f_0 = 0, \tag{2.1b}$$

and hence we could introduce the following notations

$$A_r := \langle \psi_r, (D_{xx} f_0 v_0 + D_{x\lambda} f_0)\phi_r\rangle, \tag{2.2a}$$

$$B_r := \langle \psi_r, (D_{xx} f_0 u_0 + D_{x\mu} f_0)\phi_r\rangle, r = s, a. \tag{2.2b}$$