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MULTIGRID METHODS FOR MORLEY ELEMENT ON NONNESTED MESHES*1)

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Abstract

In this paper, we consider some multigrid algorithms for the biharmonic problem discretized by Morley element on nonnested meshes. Through taking the averages of the nodal variables we construct an intergrid transfer operator that satisfies a certain stable approximation property. The so-called regularity-approximation assumption is then established. Optimal convergence properties of the W-cycle and a uniform condition number estimate for the variable V-cycle preconditioner are presented. This technique is applicable to other nonconforming plate elements.

Key words: Multigrid method, Morley element, Nonnested meshes.

1. Introduction

We consider some multigrid algorithms for the biharmonic equation discretized by Morley element on nonnested meshes. To define a multigrid algorithm, certain intergrid transfer operator has to be constructed. Through taking the averages of the nodal variables, we construct an intergrid transfer operator for Morley element on nonnested meshes that satisfies a certain stable approximation property which plays a key role in multigrid methods for nonconforming plate elements on nonnested meshes. The so-called regularity-approximation assumption is established by using the stable approximation property of the intergrid transfer operator. Optimal convergence properties of the W-cycle and a uniform condition number estimate for the variable V-cycle preconditioner are obtained by applying the abstract theory of Bramble, Pasciak and Xu [2]. This technique is applicable to other nonconforming plate elements.

There are some earlier papers on multigrid methods for nonconforming plate elements. Peisker and Braess [6] considered the $W-$cycle for the Morley element. Brenner [3] studied the W-cycle for Morley element through defining the intergrid transfer operator by taking the averages of the nodal variables and simplified the algorithms and analysis. Shi, Yu and Xie [8] studied the W-cycle for Bergan's energy-orthogonal plate

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element through defining the intergrid transfer operator by taking a linear combination of the nodal parameters of the same coarse grid element. Recently, Bramble [1] discussed variable V-cycle preconditioner for Morley element. All these papers consider the case when the triangulations are nested.

The paper is organized as follows. In section 2, we briefly describe the Morley approximation of the biharmonic Dirichlet problem. In section 3, we define an intergrid transfer operator and establish a certain stable approximation property of the intergrid transfer operator using a direct technique [9]. In section 4, we describe the multigrid methods, and establish the optimal convergence properties of the W-cycle and a uniform condition number estimate for the variable V-cycle preconditioner for Morley element on nonnested meshes.

2. Morley Element Approximation

We consider the biharmonic problem in Ω with Dirichlet boundary conditions $\Delta^2 u = f$, in Ω and $u = \dfrac{\partial u}{\partial n} = 0$, on $\partial\Omega$, where Ω is a convex polygon in R^2, $f \in H^{-l}(l = 0, 1)$.

The variational form of the problem is: Find $u \in H_0^2(\Omega)$ such that

$$a(u, v) = (f, v), \ \forall v \in H_0^2(\Omega), \tag{2.1}$$

where

$$a(u, v) = \sum_{|\alpha|=2} \int_\Omega D^\alpha u D^\alpha v dx, \ (f, v) = \int_\Omega f v dx.$$

Let $\{\Gamma_k\}, k \geq 1$, be a family of quasi-uniform triangulations of Ω. Let $h_k = \max\{\text{diam}\tau;\ \tau \in \Gamma_k\}$. We allow nonnested triangulations; however, we assume that the mesh parameters h_k satisfy $0 < \gamma_1 \leq h_{k+1}/h_k \leq \gamma_2 < 1$, where $\gamma_i (i = 1, 2)$ are constants independent of k. From this assumption we see that for $\tau \in \Gamma_k$, the number of elements $\{\tau' \in \Gamma_{k-1}$ or $\tau' \in \Gamma_{k+1};\ \bar{\tau}' \cap \tau \neq \phi\}$ is finite and is independent of k. Let V_k be Morley element space with respect to Γ_k [4,7] such that

a) for each triangle $\tau \in \Gamma_k$, $u|_\tau$ is a quadratic polynomial,

b) u is continuous at vertices and vanishes at vertices along $\partial\Omega$,

c) the normal derivative $\dfrac{\partial u}{\partial n}$ is continuous at the midpoints of each $\tau \in \Gamma_k$ and vanishes at midpoints along $\partial\Omega$.

The finite element method of the problem (2.1) is: Find $u_k \in V_k$ such that

$$a_k(u_k, v_k) = (f, v), \ \forall v \in V_k, \tag{2.2}$$

where

$$a_k(u, v) = \sum_{\tau \in \Gamma_k} \sum_{|\alpha|=2} \int_\tau D^\alpha u D^\alpha v dx.$$

Denote the induced norm $\|u\|_{2,h_k} = (a_k(u, u))^{1/2}$. Let Π_k be the nodal interpolation operator of Morley element from $H^3(\Omega) \cap H_0^2(\Omega)$ onto V_k. The following estimate for the interpolation error is known (cf.[4, 7]):

$$\|w - \Pi_k w\|_{2,h_k} \leq C h_k |w|_{H^3(\Omega)} \tag{2.3}$$

for all $w \in H^3(\Omega) \cap H_0^2(\Omega)$. Through this paper we let C (with or without subscripts)