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THE MULTIPLICATIVE COMPLEXITY AND ALGORITHM OF THE GENERALIZED DISCRETE FOURIER TRANSFORM(GFT)*

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Abstract

In this paper, we have proved that the lower bound of the number of real multiplications for computing a length 2^t real GFT(a,b) ($a = \pm 1/2, b = 0$ *or* $b = \pm 1/2, a = 0$) is $2^{t+1} - 2t - 2$ and that for computing a length 2^t real GFT(a,b)($a = \pm 1/2, b = \pm 1/2$) is $2^{t+1} - 2$. Practical algorithms which meet the lower bounds of multiplications are given.

1. Introduction

Since the fast Fourier transform was proposed, great interests for fast algorithms have been aroused. In this area, there have been many achievements, which have greatly stimulated the development of digital signal processing and other fields. The computational complexity is to study what the best algorithm will be for a given problem. There are many standards for appraising whether an algorithm is good or bad. In numerical computation, a common standard is the number of multiplications, that is, we say an algorithm is good or bad if the number of multiplications is large or small. The famous mathematican S. Winograd and L. Auslander have done some pioneering works in this area. They found the lower bound of the number of multiplications for multiplying two polynomials, and also gave an algorithm which met the lower bound[1]. Some later,they found the lower bound of the number of mulitplications for computing the discrete Fourier transform (DFT)[2−3]. After that, Heidemann-Burrus and Duhamel et al. also studied the multiplicative complexity of DFT. Heidemann-Burrus pointed out that $2^{t+1} - t^2 - t - 2$ multiplications is necessary for computing a length-2^t DFT, and also gave a practical algorithm which met the bound[4]. Some later, Duhamel et al. also proved the assertion and gave a new algorithm[5].

Generalized discrete Fourier transform (GFT) is a generalization of DFT (In ref.[10] the transform is called DET). It is shown that GFT is better than DFT in some applications. Let $x(n)(n = 0, 1, \cdots, N-1)$ be a real number sequence, we call

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{(n+a)(k+b)}, k = 0, 1, \cdots, N-1. \tag{1}$$

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the generalized discrete Fourier transform of $\{x(n)\}$, where $W_N = e^{-i\frac{2\pi}{N}}$ and $i = \sqrt{-1}$. In (1) a is called the time parameter and b the frequency parameter. A GFT with time parameter a and frequency parameter b is denoted by GFT(a,b). Especially, if $a = b = 0$, (1) is the DFT. It is very interesting to determine the multiplicative complexity of GFT. Since in practical applications a,b can either be 0 or $\pm 1/2$, so when we discuss the multiplicative complexity, we confine our research on these cases.

2. The Computation of GFT(a,b)(a,b are integers)

If a,b are integers, the compution of GFT(a,b) is almost the same as that of DFT. In fact, if we set

$$x^{'}(n) = \begin{cases} x(N+n-a), & n = 0, 1, \cdots, a-1 \\ x(n-a), & n = a, \cdots, N-1 \end{cases}$$

and denote the DFT of $\{x^{'}(n)\}$ by $\{X^{'}(k)\}$, then it is easy to prove that

$$X(k-b) = X^{'}(k), \quad k \in Z$$

where $\{X(k)\}$ means the GFT(a,b) of $\{x(n)\}$. Therefore, the multiplicative complexity of GFT(a,b)(a,b are integers) is the same as that of DFT.

3. The Multiplicative Complexity and Algorithm of GFT(0,1/2) and GFT(1/2,0)

1. The relationship between DFT and GFT(1/2,0)

Let $\{Y(k)\}$ be the DFT of $\{x(n)\}(n = 0, 1, \cdots, N-1; N = 2^t)$, that is

$$Y(k) = \sum_{n=0}^{N-1} x(n) W_N^{nk}, k = 0, 1, \cdots, N-1.$$

In the following, $\{Y(k)\}$ is turned to a series of GFT(1/2,0).

$$Y(k) = \sum_{n=0}^{N/2-1} x(2n) W_{N/2}^{nk} + \sum_{n=0}^{N/2-1} x(2n+1) W_{N/2}^{(n+1/2)k} .$$

If we set $\{U(k)\}$ and $\{V(k)\}$ to be the DFT of $\{x(2n)\}(n = 0, 1, \cdots, N/2-1)$ and the GFT(1/2,0) of $\{x(2n+1)\}(n = 0, 1, \cdots, N/2-1)$ respectively, then

$$Y(k) = U(k) + V(k), k = 0, 1, \cdots, N/2-1,$$

$$Y(k+N/2) = U(k) - V(k), k = 0, 1, \cdots, N/2-1.$$

Therefore, a DFT of length N is decomposed to a DFT of length N/2 and a GFT(1/2,0) of length N/2 plus N additions. If $N/2 \geq 2$, the decomposition can continue. In