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A MODIFIED CONJUGATE DIRECTION METHOD FOR COMPUTING THE PSEUDOINVERSE*

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Abstract

In this paper we are concerned with the modified conjugate direction method for computing the pseudoinverse by using an orthogonal basis of the range space of A. Numerical results show that the new method retains some main advantages in terms of efficiency and accuracy.

1. Introduction

The method of least squares is a standard tool for solving problems such as control, state evaluation and identification[2]. The linear least square problem is defined as the minimization of the norm of the residual vector

$$\min_x \|Ax - b\|_2^2, \tag{1}$$

where $A \in R^{m\times n}$ with rank k, $b \in R^m$ is a real vector to be approximated, and $x \in R^n$ is a real vector.

Connecting with the linear least squares problem (1), the computation of the pseudoinverse of A is also quite common in this context. A real $n \times m$ matrix G is called the pseudoinverse of A if G satisfies the following conditions:

$$(1)AGA = A, \quad (2)GAG = G, \quad (3)(AG)^T = AG, \quad (4)(GA)^T = GA, \tag{2}$$

and can be written as

$$A^+ = G.$$

Thus, the least squares solution of the minimum norm of problem (1) is

$$x = A^+b. \tag{3}$$

This solution is unique whether the problem is consistent or not.

In this paper, a class of conjugate direction method for computing the pseudoinverse is considered. The given method requires less computational work and has other advantages.

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Throughout this paper, let $R(\cdot), N(\cdot)$ stand for the range space and the null space of a matrix, respectively. $R(x)^{\perp}$ denotes an orthogonal complement of $R(\cdot)$ and $\|\cdot\|$ the Euclidian vector norm.

2. The Case of the Full Rank Matrix

In this section we are concerned only with the simplest case that A is an $m \times m$ symmetric positive definite matrix. Let vectors $p_1, p_2, \cdots, p_m \in R^m$ be mutually conjugate, i.e.

$$p_i^T A p_j = \begin{cases} 0, & i \neq j, \\ d_i \neq 0, & i = j. \end{cases}$$

Using these vectors, we can easily obtain the sequence of matrices

$$G_i = \sum_{j=1}^{i} \frac{p_j p_j^T}{d_j}, \quad i = 1, 2, \cdots, m.$$

Due to the conjugacy of vectors $p_1, p_2, \cdots, p_m$, we have

$$G_i A p_j = p_j, \quad j \leq i, \tag{4}$$

and

$$G_i A p_j = 0, \quad j > i. \tag{5}$$

In particular, for $i = m$, the matrix G_m satisfies

$$G_m A p_j = p_j \tag{6}$$

or

$$(G_m A - I) p_j = 0, \quad j = 1, 2, \cdots, m.$$

Since vectors $p_1, p_2, \cdots, p_m$ are linearly independent, it follows that

$$G_m A = I$$

and

$$G_m = A^{-1}.$$

Observe that the sequence of matrices $G_i, i = 1, 2, \cdots, m$, are generated by the following relation:

$$G_0 = 0, \quad G_i = G_{i-1} + \frac{p_i p_i^T}{d_i}. \tag{7}$$

Summarizing, we have[3]

Theorem 1. *Let $A \in R^{m \times m}$ be a symmetric positive definite matrix. Given a set of vectors $p_1, p_2, \cdots, p_m \in R^m$, which are mutually conjugate, and the matrices $G_0, G_1, \cdots, G_m \in R^{m \times m}$ generated by the recurrence relation (7), then $G_i, i =$*