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# CHEBYSHEV APPROXIMATION OF THE ANALYTICAL SOLUTION OF DIRICHLET PROBLEM*

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**Abstract**

In this paper linear programming method for minimax approximation is used to obtain an approximation to the analytical solution of a Dirichlet problem using the logarithmic potential function as an approximating function. This approach has the advantage of producing a better approximation than that using other solution of the potential equation as an approximating or basis function for a problem in $n = 2$ dimensions.

## §1. Introduction

It has been stated that a problem involving the position of a point in space may be regarded as two dimensional whenever it may be made to depend on two real coordinates. An infinite straight wire of constant density $\lambda$ produces such a field. The attraction is perpendicular to it and it is in accordance with the law of the inverse first power of the distance. Its magnitude in attraction units is $2\lambda/r$ where $r$ is the distance of the attracted unit particle from the wire. The potential of such a particle is $2\lambda\log(1/r)$ where the constant which may be added to the potential has been determined so that the potential vanishes at a unit distance from the particle.

It is known that continuous distributions of matter attracting according to the law of inverse first power are interpretable as distributions of matter attracting according to Newton's law on infinite cylinders or throughout volumes bounded by infinite cylinders whose densities are the same at all points of the generators of the cylinders or of lines parallel to them. Since the total mass of such a cylinder does not vanish, its potential cannot vanish at infinity. It can only become infinite. In order to make the zero of the potential to be defined, it is made to vanish at a unit distance from the attracted particle in the case of a particle, and, in the case of a continuous distribution, by integrating the potential of a unit particle multiplied by the density over the curve or area occupied by matter. The potential is then defined by the integrals

$$u = \int_C \lambda\log(1/r)ds, \quad u = \int_A\int \sigma\log(1/r)ds \tag{1.1}$$

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for distributions on curves and over areas respectively.

These potentials which are regarded as plane material curves or plane laminas whose elements attract according to the law of the inverse first power are distinguished from the potentials of curves and laminas whose elements attract according to Newton's law by calling them the logarithmic potentials.

Logarithmic potentials are the limiting forms of Newtonian potentials[6].

**A Unit Source and Principal Solution**

Assuming that the discontinuity of $u$ at $Q$ consists of a unit source, that is, the yield $q$ of a source $Q$ is defined as the outward gradient of its field $u$, and denoting the distance from $Q$ by $\rho$, we have[11]

$$q = \int_K \frac{\partial u}{\partial \rho} ds \tag{1.2}$$

where $K$ is a circle of arbitrarily small radius about $Q$. Assuming that in the immediate neighbourhood of the source, $u$ depends only on $\rho$, then we get by transformation

$$q = \int_{-\pi}^{\pi} \frac{du}{d\rho} \rho d\psi = 2\pi\rho \frac{du}{d\rho} \tag{1.3}$$

A unit source is therefore given by

$$1 = 2\pi\rho \frac{du}{d\rho} \quad \text{or} \quad \frac{du}{d\rho} = \frac{1}{2\pi\rho} \tag{1.4}$$

and

$$u = \frac{1}{2\pi} \log \rho + \text{constant} \quad \text{for} \quad \rho \to 0. \tag{1.5}$$

For arbitrary $\rho$, we have

$$u = U \log \rho + V, \quad \rho = \{(x-\xi)^2 + (y-\eta)^2\}^{1/2} \tag{1.6}$$

where $U$ and $V$ are analytic funcitons of $(x, y)$ and $(\xi, \eta)$ such that $U$ becomes $1/2\pi$ when $\rho \to 0$.

A solution like (1.6) is known as the principal solution of the differential equation $M(u) = 0$ where

$$M(u) = \frac{\partial^2 Au}{\partial x^2} + 2\frac{\partial^2 Bu}{\partial x \partial y} + \frac{\partial^2 Cu}{\partial y^2} - \frac{\partial Du}{\partial x} - \frac{\partial Eu}{\partial y} + Fu. \tag{1.7}$$

In the case of the potential equaiton $\Delta u = 0$, the principal solution corresponds to the logarithmic potential

$$u = \frac{1}{2\pi} \log \rho \ \text{ for all } \rho. \tag{1.8}$$

For the three space-dimensional case, the analogue of (1.4) is

$$\frac{\partial u}{\partial \rho} = \frac{1}{4\pi\rho^2}, \quad u = \frac{1}{4\pi\rho} + \text{constant} \tag{1.9}$$

where $\rho$ is as defined previously and $4\pi\rho^2$ is the surface area of the sphere of radius $\rho$ enclosing $Q$.