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FURTHER DEVELOPMENTS IN AN ITERATIVE PROJECTION AND CONTRACTION METHOD FOR LINEAR PROGRAMMING*1)

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Abstract

A linear programming problem can be translated into an equivalent general linear complementarity problem, which can be solved by an iterative projection and contraction (PC) method [6]. The PC method requires only two matrix-vector multiplications at each iteration and the efficiency in practice usually depends on the sparsity of the constraint-matrix. The prime PC algorithm in [6] is globally convergent; however, no statement can be made about the rate of convergence. Although a variant of the PC algorithm with constant step-size for linear programming [7] has a linear speed of convergence, it converges much slower in practice than the prime method [6]. In this paper, we develop a new step-size rule for the PC algorithm for linear programming such that the resulting algorithm is globally linearly convergent. We present some numerical experiments to indicate that it also works better in practice than the prime algorithm.

1. Introduction

This paper presents an algorithm for linear programming problems based on an iterative projection and contraction method for linear complementarity problems [6]. The algorithm makes a trivial projection onto a general orthant at each iteration and the generated sequence contracts Féjer-monotonically to the solution set, i.e., the Euclidean distance of the iterates to the solution set decreases at each iteration. Usually the matrices describing the constraints for large problems will be sparse, but often no special structure pattern is detectable in it. The projection and contraction method for linear complementarity problems is an iterative procedure which requires in each step only two matrix-vector multiplications, and performs no transformation on the matrix elements. The method therefore allows the optimal exploitation of the sparsity of the constraint matrices and may thus be an efficient method for large sparse problems ([6] and [7]).

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In this paper, we work on the ideas in [6] and [7], and obtain a faster algorithm. The directions generated by this algorithm are the same as generated by the algorithms presented in [6] and [7]. However, we get a new simple step-size rule and are able to obtain global linear convergence without estimation of the norm of the matrix describing the constraints. Moreover, the new algorithm also works better in practice than the prime algorithm.

Our paper is organized as follows. In Section 2, we quote some theoretical background from [6]. Section 3 describes the new algorithm and its relation to other PC algorithms. Section 4 proves the convergence properties of our new algorithm. Section 5 gives an extension–the scaled algorithm. In Section 6, we present some numerical results. Finally, in Section 7, we conclude the paper with some remarks.

We use the same notations as in [6]. The i-th component of a vector x in the real n-dimensional Euclidean space R^n is denoted by x_i. A superscript such as in u^k refers to specific vectors and k usually denotes the iteration index. $P_\Omega(\cdot)$ denotes the orthogonal projection on the convex closed set Ω. Specifically, x_+ denotes the projection of x on nonnegative orthant R^n_+, i.e.,

$$(x_+)_i := \max\{0, x_i\}, \qquad i = 1, \ldots, n.$$

$\|\cdot\|$ and $\|\cdot\|_\infty$ denote the Euclidean and the max-norm, respectively. For a positive definite matrix G, the norm $\|u\|_G$ is given by $(u^T G u)^{\frac{1}{2}}$.

2. Theoretical Background

We consider the pair of the standard form linear program and its dual

$$\text{(P)} \qquad \begin{aligned} &\min \quad c^T x, \\ &\text{s.t} \quad Ax = b, \quad x \geq 0, \end{aligned} \qquad (1)$$

$$\text{(D)} \qquad \begin{aligned} &\max \quad b^T y, \\ &\text{s.t} \quad A^T y \leq c \end{aligned} \qquad (2)$$

where A is an $m \times n$-matrix and b, c are vectors of length m and n, respectively. Let

$$\Omega^* := \{u = (x, y) | x \text{ is a solution of (P)}, \ y \text{ is a solution of (D)}\}. \qquad (3)$$

Throughout the paper we assume that $\Omega^* \neq \emptyset$. It is well known [2] that $u = (x, y) \in \Omega^*$ if and only if it solves the following general linear complementarity problem:

$$\text{(LCP)} \qquad \begin{cases} x \geq 0, \quad -A^T y + c \geq 0, \quad x^T(-A^T y + c) = 0, \\ Ax - b = 0. \end{cases} \qquad (4)$$

Let

$$M := \begin{pmatrix} & -A^T \\ A & \end{pmatrix}, \qquad q := \begin{pmatrix} c \\ -b \end{pmatrix}, \qquad \Omega := \{u = (x, y) | x \geq 0\}. \qquad (5)$$