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NON-CONFORMING DOMAIN DECOMPOSITION WITH HYBRID METHOD*[1)]

Liang Gou-ping Liang Ping
(*Institute of Mathematics, Academia Sinica, Beijing, China*)

Abstract

We present a non-conforming domain decomposition technique for solving elliptic problems with the finite element method. Functions in the finite element space associated with this method may be discontinuous on the boundary of subdomains. The sizes of the finite meshes, the kinds of elements and the kinds of interpolation functions may be different in different subdomains. So, this method is more convenient and more efficient than the conforming domain decomposition method. We prove that the solution obtained by this method has the same convergence rate as by the conforming method, and both the condition number and the order of the capacitance matrix are much lower than those in the conforming case.

§1. Introduction

Along with the development of the parallel computer in recent years, there has been a growing interest in methods based on domain decomposition for the numerical solution of elliptic partial differential equations. The key idea of this method is that the domain of the problem is decomposed into smaller subdomains, and then a computer is used to solve the problem on each subdomain. This is an efficient method for solving the big problem of elliptic partial differential equations on the parallel computer.

Up to now, there are only conforming finite elements with domain decomposition methods, with which the function of the finite element space must be compatible on the whole domain of the problem. However, it will be more convenient and more efficient to adopt different sizes of meshes and different kinds of shape functions in different subdomains when solving practical problems in science and engineering. But this is impossible for conforming finite elements.

The aim of this paper is to put forward a non-conforming domain decomposition for elliptic problems. This method needs no compatibility on the boundary of subdomains, that is to say, the function of the finite element space may be discontinuous on the boundary of subdomains. With this property we can use different sizes of meshes, different kinds of elements in different subdomains. We will prove that the convergence rate of the solution obtained by this method is the same as by the conforming method; moreover, the condition number and the order of the capacitance matrix are much lower than in the conforming case. In this paper, we only consider the method itself as well as the error and the condition number estimates. Solution by this method of the algebraic system of equations, which arises from the discretization of elliptic equations, will be discussed in another paper.

In Section2, we will introduce the decomposition of the domain and the construction of the finite element space. Section 3 contains the non-conforming method and the matrix

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representation. The error estimate of the energy norm will be obtained in Section 4. Finally, in Section 5 the condition number of the capacitance matrix will be given.

§2. The Decomposition of the Domain and Finite Element Space

For simplicity, we only consider the Dirichlet problem for the Poisson equation

$$\begin{cases} -\Delta u = f & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega. \end{cases} \tag{2.1}$$

We suppose that the domain Ω is a polygon.

We first decompose the domain Ω into subdomains Ω_i; then we subdivide the subdomain Ω_i and its boundary into finite elements.

More precisely, we shall begin with the following assumption with regard to Ω.

A1: Ω is a polygonal domain.

A2: For each $d, d > 0$, as a parameter, the domain Ω is decomposed into quasi-uniform subdomains $\Omega_i (i = 1, 2, \cdots, n)$ with size d. By this we mean that there exists a positive constant c independent of d such that each subdomain Ω_i contains a ball of diameter cd and is contained in ball of diameter d.

A3: For each parameter h, $0 < h < d$, the subdomain Ω_i is subdivided into quasi-uniform finite elements with size h. The meaning of this assumption is as above.

Let $\Omega_i^{h_i}$ be the union set of all elements in Ω_i, and $\Omega^h = \bigcup_i \Omega_i^{h_i}$.

A4: Let Γ be the union set of all boundaries of the subdomains, that is $\Gamma = \bigcup_i \partial\Omega_i$. For each $H, 0 < h < H \leq d, \Gamma$ is subdivided into quasi-uniform line segments with size H. Its meaning is similar to A2. The vertices of Ω_i must be the vertices of elements. Let Γ^H be the union set of all line segments in Γ.

We always suppose that $0 < h < H \leq d$ and assume the asymptotic behavior

$$\lim_{h \to 0} \frac{h}{H} = 0 \tag{2.2}$$

where $h = \max_i (h_i)$.

Completing the decomposition of the domain, we now construct the space of the finite elements. We make the following supposition:

Let $S_{h_i}(\Omega_i)$ be the space of piecewise m-th polynomial functions which are continuously defined in subdomain $\Omega_i^{h_i}$ and vanish on $\partial\Omega \cap \partial\Omega_i$.

Let $S_h^0(\Omega)$ be the space of functions defined in $\Omega^h = \bigcup_i \Omega_i^{h_i}$, which are continuous and piecewise m-th polynomials in $\Omega_i^{h_i}$ and vanish on $\partial\Omega$. We emphasize that the functions in $S_h^0(\Omega)$ are only continuous in Ω_i but may be discontinuous on Ω.

Let $S_H(\Gamma)$ be the space of piecewise n-th polynomial functions continuously defined on Γ^H and vanishing on $\partial\Omega$. $S_H(\partial\Omega_i)$ is the space of piecewise n-th polynomial functions continuously defined on $\Gamma^H \cap \partial\Omega_i$ and vanishing on $\partial\Omega \cap \partial\Omega_i$.

We define the finite element space $S_{h\times H}$ as follows:

$S_{h\times H} \subset S_h^0(\Omega) \times S_H(\Gamma)$, $(u, \varphi) \in S_{h\times H}$ is and only if $(u, \varphi) \in S_h^0(\Omega) \times S_H(\Gamma)$ and $u = \varphi$ on the nodes emerging during subdividing the subdomain Ω_i into elements.

Space $S_{h\times H}$ is a subspace of $S_h^0(\Omega) \times S_h(\Gamma)$. It can be easily seen that $S_h(\Gamma) = (\varphi | (u, \varphi) \in S_{h\times H})$.