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A NUMERICAL METHOD OF THE RAMM INTEGRAL EQUATION *

Tang Long-ji Xie Gan-quan

(Hunan Computing Center, Changsha, Hunan, China)

Abstract

A numerical method for solving the ill-posed Ramm integral equation is presented in this paper. It is found that the method is stable and more accurate. Particularly, when the given data is contaminated by noise, satisfactory results are obtained by using the algorithm of this paper.

§1. Introduction

In this paper we consider the numerical solution of the following Fredholm first kind integral equation which was derived by A.G.Ramm in [1], [2]:

$$\int_B \frac{V(z)}{|x'-z|\,|y'-z|}dz = f(x',y') \tag{1.1}$$

where

$$B=\Big\{z\big|z\in R^3,|z-z_0|\le d,z_0=(0,0,-a),a>d\Big\},$$

$$P=\Big\{z\big|z=(z_1,z_2,z_3)\in R^3,z_3=0\Big\},$$

$$x'=(x_1,x_2)\in P,\quad y'=(y_1,y_2)\in P,$$

$f(x',y')$ is the datum which can be measured, $V(z)$ is the wanted solution,

$$K(z;x',y')=\frac{1}{|x'-z|\,|y'-z|}$$

is the kernel function of (1.1). Clearly, $K(z;x',y')\in C^\infty(B)$, so equation (1.1) is ill-posed. We will present an efficient numerical method for solving (1.1) and give satisfactory computing results.

§2. The Numerical Method

2.1. Super-Isoparametric Finite Element [4] Discretization of (1.1)

Now we present the super-isoparametric finite element discretization scheme for the numerical solution of equation (1.1). The method is as follows.

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Subdivide B into a set of nonoverlapping hexahedron $e_i, i = 1, 2, \cdots, p$, i.e. $B = \bigcup_{i=1}^{p} e_i, e_i \bigcap e_j = \phi, i \neq j$. The subdivision points are $z_1, z_2, \cdots, z_n$. And get a parametric mapping for each element

$$e_i \longleftrightarrow \hat{e}. \tag{2.1}$$

$$z_1 = f_1(\xi, \eta, \varsigma), \quad z_2 = f_2(\xi, \eta, \varsigma), \quad z_3 = f_3(\xi, \eta, \varsigma), \tag{2.2}$$

where $\hat{e}$ is the standard element–cube, and f_1, f_2, f_3 are polynomials. We find that, for the ill-posed problem, a very wonderful form of element mapping is the so-called super-isoparametric element in which the order of the mapping functions $f_i, i = 1, 2, 3$, is greater than that of the shape functions in $\hat{e}$. Here, we use 20-node two order serendiping-type interpolation for coordinates (z_1, z_2, z_3), and use 8-node trilinear interpolation for the unknown function V. If $\phi_j^e(\xi, \eta, \varsigma)$ represents a standard type of the finite element shape function for a 20–node element in the local domain [4], we can write the mapping relationship of (2.2) for each element as

$$z_1 = \sum_{j=1}^{20} z_{1j}\phi_j^e, \tag{2.3}$$

$$z_2 = \sum_{j=1}^{20} z_{2j}\phi_j^e, \tag{2.4}$$

$$z_3 = \sum_{j=1}^{20} z_{3j}\phi_j^e, \tag{2.5}$$

in which $\phi_j^e = \phi_j^e(\xi, \eta, \varsigma)$.

Let $\psi_j^e(\xi, \eta, \varsigma), j = 1, 2, \cdots, 8$, be a system of trilinear interpolation base function[4] in the local domain. Then the finite element approximate solution of V in $\hat{e}$ can be written in the following form

$$V^h = \sum_{j=1}^{8} V_j \Psi_j^e(\xi, \eta, \varsigma). \tag{2.6}$$

For given $x_i' = (x_{1i}, x_{2i}), y_i' = (y_{1i}, y_{2i})$, we have

$$\int_B K(z; x_i', y_i')V(z)dz = \sum_{l=1}^{p} \int_{e_l} K(z; x_i', y_i')V(z)dz. \tag{2.7}$$

In any element e_l,

$$\begin{aligned} \int_{e_l} K(z; x_i', y_i')V(z)dz &\sim \int_{\hat{e}} K(\xi, \eta, \varsigma; x_i', y_i') \sum_j V_j \Psi_j^e |J| d\xi d\eta d\varsigma \\ &= \sum_j \Big(\int_{\hat{e}} K(\xi, \eta, \varsigma; x_i', y_i') \Psi_j^e |J| d\xi d\eta d\varsigma \Big) V_j = \sum_j a_{ij}^{e_l} V_j \end{aligned} \tag{2.8}$$