NUMERICAL TESTS ON CONVERGENCE OF THE RANDOM CHOICE METHOD*1)

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The random choice method (RCM) has been successfully used for computing very complicated combustion problems in [1], which shows its robustness. In this paper, we shall observe its convergence through numerical tests.

The problem computed in this paper is the ignition problem. The formulation of the problem and the RCM method can be found in [1].

We have computed this problem using five different meshes and two different sequences of random numbers, and estimated the error of pressure obtained by RCM in L_2 norm, defined by

$$\sigma=\left[\frac{1}{b-a}\int_a^b [p(x, t_0)-p^*(x, t_0)]^2 dx\right]^{1/2},$$

where $p(x, t_0)$ denotes the approximate pressure at time $t=t_0$ and $p^*(x, t_0)$ the exact one, and $[a, b]$ is the computational interval in the x-direction. In our computation, the numbers of mesh points in the x-direction are 81, 161, 321, 641, 1281. In Tables 1 and 2 the values of σ for $t=3$ and 11 are given. We can find from the tables that the results possess strong randomness. The values of σ change by $20\%\sim50\%$ when different sequences of random numbers are adopted. Moreover, the error is not a monotonic decreasing function, though the general trend of error is on the decrease while Δt decreases.

In what follows we shall make a rough estimate of convergence rate using the data in the tables. Suppose that the rate of convergence is $O(\Delta t^a)$. Therefore between the error and the parameter a there is the following approximate relation

$$\frac{\Delta t_1^a}{\Delta t_2^a}=\frac{\sigma(\Delta t_1)}{\sigma(\Delta t_2)},$$

Table 1 Errors of RCM for $t=3.0$

Numbers of mesh points	Errors σ_1 (sequence 1)	Errors σ_2 (sequence 2)
81	0.180	0.234
161	0.176	0.127
321	0.127	0.055
641	0.065	0.0635
1281	0.078	0.0408

* Received March 27, 1985.
1) Projects Supported by the Science Fund of the Chinese Academy of Sciences.

Table 2 Errors of RCM for $t=11.0$

Numbers of mesh points	Errors σ_1 (sequence 1)	Errors σ_2 (sequence 2)
81	4.48	5.50
161	4.67	5.21
321	2.36	3.96
641	3.43	2.01
1281	3.27	3.08

Because this problem is very complicated, no analytical solution has been obtained. The solution obtained by using the Singularity-Separating Method is quite accurate. It was taken as the exact solution while we computed σ. This substitution will not have an essential influence on the correctness of the values in Tables 1 and 2 since the error of RCM is much larger than that of the Singularity-Separating Method.

which can be rewritten as

$$a=\frac{\log\frac{\sigma(\Delta t_1)}{\sigma(\Delta t_2)}}{\log\frac{\Delta t_1}{\Delta t_2}}=\frac{\log\frac{\sigma(\Delta t_1)}{\sigma(\Delta t_2)}}{\log\frac{\Delta x_1}{\Delta x_2}}.$$

Here Δt denotes the step size in the t-direction and Δx the step size in the x-direction. In our computation $\Delta x/\Delta t$ is unchanged as Δt changes. That is, we always take $\Delta t_1/\Delta t_2=\Delta x_1/\Delta x_2$, where Δt_1, Δx_1 are the two increments for a net and Δt_2, Δx_2 for the other. Generally speaking, σ depends on Δx and Δt. In our case, $\Delta x/\Delta t$ is fixed; so we could think that σ depends just on Δt. This is why we use the symbol $\sigma(\Delta t)$ instead of $\sigma(\Delta x, \Delta t)$. From Table 1 we know that while 161 points are taken in the x-direction and the first sequence of random numbers is used, $\sigma=0.176$ for $t=3$. And $\sigma=0.127$ if 321 points are taken. Therefore we have

$$a=\frac{\log\frac{0.176}{0.127}}{\log\frac{1/160}{1/320}}\approx 0.48.$$

It can be easily found that we shall obtain another approximate value of a if taking two other nets. Therefore we should compute its average. In the case $t=3$, its average is 0.47. According to Table 2, the average of a is 0.16 in the case $t=11$. Therefore, it seems that for the problem considered the convergence rate of RCM is less than $O(\Delta t^{1/2})$.

Table 3 CPU times of RCM (from $t=0$ to $t=12$)

Numbers of mesh points in the x-direction	81	161	321	641	1281
CPU times (sec)	14	36	119	367	1511

In Table 3 we list the CPU times of RCM for five different nets. As is well-known, the CPU time spent on solving a problem can be roughly divided into two parts. One part (for example, the time spent on compilation) does not depend on the total number of mesh points and the other part does. For explicit schemes, the latter is directly proportional to the total number of mesh points. In our computation